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# Enhanced Quality Assurance: New Chain Sampling Plans Using Minimum Angle Method for Generalized Exponential Distribution

(Jaminan Kualiti yang Dipertingkatkan: Pelan Pensampelan Rantaian Baharu Menggunakan Kaedah Sudut Minimum untuk Taburan Eksponen Umum)

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ABSTRACT

Acceptance sampling plays a crucial role in ensuring product quality by inspecting only a small representative sample from a large production lot. Every acceptance sampling plan outlines specific acceptance criteria that govern the decision to accept or reject the entire production lot. Chain sampling plan (ChSP-1), which is characterized by five acceptance criteria, is renowned for its leniency towards producers. On the other hand, modified chain sampling plan (MChSP-1) employs only three acceptance criteria and tends to favour the consumers. New chain sampling plan (NChSP-1) is developed in this study to strategically bridge the gap between these two existing plans. It adopts a balanced strategy by utilizing four acceptance criteria. Minimum angle method (MAM) is implemented in designing NChSP-1 through applying geometric approach that seeks to minimize the divergence between the observed and ideal operating characteristics (OC) curve. The method determines the minimum achievable angle by employing the tangent function across various quality levels. The efficacy of the sampling plan is evaluated through two significant performance parameters, which are the optimal number of sample size and the minimum angle. Both performance parameters are derived based on pre-determined design parameters related to the probability of lot acceptance and the lifetime distribution, where generalized exponential (GE) distribution is used to model the lifetime in this study. Comparative analysis shows that NChSP-1 consistently produces smaller optimal sample size and minimum angle than ChSP-1, indicating its advantages in risk balancing and resource efficiency. These outcomes highlight the applicability of NChSP-1 for industrial practitioners engaged in life testing procedures.

Keywords: Acceptance criteria; minimum angle; new chain sampling

ABSTRAK

Pensampelan penerimaan memainkan peranan penting dalam memastikan kualiti produk dengan memeriksa hanya sampel perwakilan yang kecil daripada lot pengeluaran yang besar. Setiap pelan persampelan penerimaan menggariskan kriteria penerimaan khusus yang mengawal keputusan untuk menerima atau menolak keseluruhan lot pengeluaran. Pelan persampelan berantai (ChSP-1) yang dicirikan oleh lima kriteria penerimaan terkenal dengan kelonggarannya terhadap pengeluar. Sebaliknya, pelan persampelan berantai terubah suai (MChSP-1) hanya menggunakan tiga kriteria penerimaan dan cenderung memihak kepada pengguna. Pelan persampelan berantai baharu (NChSP-1) dibangunkan dalam kajian ini untuk merapatkan jurang secara strategik antara dua pelan sedia ada ini. Ia mengguna pakai strategi yang seimbang dengan memanfaatkan empat kriteria penerimaan. Kaedah sudut minimum (MAM) dilaksanakan dalam mereka bentuk NChSP-1 melalui pendekatan geometri yang bertujuan untuk meminimumkan sisihan antara lengkung cirian pengoperasian (OC) yang diperhatikan dan yang ideal. Kaedah ini menentukan sudut minimum yang boleh dicapai dengan menggunakan fungsi tangen merentasi pelbagai tahap kualiti. Kelebihan pelan persampelan ini dinilai melalui dua parameter prestasi penting, iaitu saiz sampel optimum dan sudut minimum. Kedua-dua parameter prestasi ini diperoleh berdasarkan parameter reka bentuk yang telah ditetapkan berkaitan dengan kebarangkalian penerimaan lot dan taburan jangka hayat dengan taburan eksponen teritlak (GE) digunakan untuk memodelkan jangka hayat dalam kajian ini. Analisis perbandingan menunjukkan bahawa NChSP-1 secara tekal menghasilkan saiz sampel optimum dan sudut minimum yang lebih kecil berbanding ChSP-1, yang menunjukkan kelebihannya dalam pengimbangan risiko dan kecekapan sumber. Hasil ini menonjolkan kebolehgunaan NChSP-1 untuk pengamal industri yang terlibat dalam prosedur pengujian jangka hayat.

Kata kunci: Kriteria penerimaan; pensampelan berantai baharu; sudut minimum

## INTRODUCTION

In the realm of quality assurance, acceptance sampling serves as a vital statistical tool used to determine whether a batch of products meets established standards. Initially developed during World War II by Harold F. Dodge, acceptance sampling was formalized through the MIL-STD-105 framework (Schilling & Neubauer 2009), laying the groundwork for modern quality inspection protocols. Today, its relevance spans industries such as manufacturing, healthcare, defense, and construction, where reliable product assessment is crucial for operational and economic efficiency. As outlined by Montgomery (2012), acceptance sampling supports decision-making by inspecting a representative sample to determine the quality status of the entire batch, which can be particularly beneficial in large-scale production settings or when testing is naturally destructive. This process allows organizations to make statistically informed decisions about product quality while balancing between cost efficiency and quality assurance.

Acceptance sampling operates under two key risks: the producer's risk (Type I error), which involves the rejection of a lot that meets quality standards and the consumer's risk (Type II error), which arises when a defective lot is incorrectly accepted (Montgomery 2012). These risks can have serious implications for both parties involved, affecting business relationships, product liability and operational reputation. Acceptance sampling plans are designed to manage these risks by establishing acceptable levels for each.

Depending on the context, acceptance sampling plans may be based on variable sampling, which uses precise numerical data or attribute sampling, which classifies items based on categorical criteria such as 'defective' or 'non-defective'. This study focuses on attribute-based acceptance sampling method due to its simplicity and practicality as it has been widely applied in various industries (Dumičić & Žmuk 2012).

A critical component of any acceptance sampling plan is the Operating Characteristic (OC) function, denoted by $L(p)$. Introduced by Dodge and Romig (1929), this function expresses the probability of accepting a lot as a function of the actual proportion of defective items it contains. The OC curve visually illustrates this relationship, showing how likely a lot is to be accepted at different quality levels. It serves as a powerful tool for both producers and consumers, enabling informed decisions about the adequacy of a lot based on acceptable risk thresholds (Chorafas 2013).

In practical scenarios where resources or time constraints hinder prolonged testing, the time-truncated life test emerges as an efficient and pragmatic solution. This method involves observing items over a fixed testing duration and terminating the test at the end of that specified period regardless of how many failures have occurred (Aslam & Ali 2019). The approach enables analysts to draw conclusions within a controlled timeframe, which is particularly beneficial in terms of cost minimization (Tsai, Jiang & Lio 2015), especially when testing is expensive, time-sensitive or when product lifetimes are lengthy. Many researchers including Adeoti and Rao (2021), Al-Momani and Al-Masri (2025), Jamaludin, Zain and Aziz (2016), Kiapour, Tripathi and Masoumi (2025), Nazrina et al. (2020), Nwankwo et al. (2024), Qomi and Fernandez (2024), Rather et al. (2025) and Wu, Wang and Chen (2025) have explored time-truncated life test applications within acceptance sampling contexts in their study.

Among the various acceptance sampling procedures, the Chain Sampling Plan (ChSP-1) proposed by Dodge (1955) is widely recognized for incorporating information from preceding lots when determining the acceptance decision for the current lot. By utilizing inspection outcomes from previous lots, ChSP-1 improves the discriminatory capability of the sampling procedure compared with conventional Single Sampling Plan (SSP). However, the plan is often considered relatively lenient towards producers because it employs a larger number of acceptance criteria. To address this limitation, the Modified Chain Sampling Plan (MChSP-1) was later introduced with stricter acceptance conditions that enhance consumer protection.

Motivated by the need to achieve a better balance between producer's and consumer's risks in life-testing inspection, this study develops a New Chain Sampling Plan denoted as NChSP-1. Existing chain sampling approaches such as ChSP-1 and MChSP-1 differ in the level of stringency imposed by their acceptance criteria. ChSP-1 is generally regarded as more lenient towards producers, whereas MChSP-1 imposes stricter acceptance conditions that favor consumer protection. These contrasting characteristics highlight the need for an alternative sampling strategy that can provide a more balanced rule for lot acceptance decision. Accordingly, NChSP-1 adopts four acceptance criteria, representing an intermediate level of stringency intended to balance the leniency of ChSP-1 and the strictness of MChSP-1. The design of the proposed plan is based on the minimum angle method (MAM), which evaluates the closeness of the operating characteristic (OC) curve to an ideal OC curve.

## LITERATURE REVIEW

The development of sampling plans has evolved significantly over time, beginning with the foundational Single Sampling Plan (SSP) as introduced by Epstein (1954). It is defined by two primary parameters: the sample size to be drawn from the lot, and the acceptance number which specifies the maximum number of defective items allowed for the lot to be accepted. Its simplicity suits small or low-variation production and has attracted many researchers to develop SSP such as Ahmed, Ali and Yousof (2024), Kadir and Rahi (2023) and Sadeghpour Gildeh, Baloui Jamkhaneh and Yari (2011).

However, if the acceptance number is set to be very low such as zero or one, even a slight increase in the defective rate can lead to an abrupt decline in the chance of accepting a lot. This is unfair burden on producers (Montgomery 2020). SSP typically requires a very large sample size to achieve the desired levels of producer's and consumer's risk, that may be impractical and economically unviable when the products under examination are expensive or involve destructive testing (Govindaraju & Lai 1998). Another major limitation of SSP is its rigid acceptance criteria, which does not consider useful performance information of the earlier lots (Stephens & Dodge 1976). This limitation may lead to diminished efficacy in differentiating between high and low-quality product lots (Balamurali & Usha 2013). Dodge (1955) proposed the Chain Sampling Plan (ChSP-1) to deal with these limitations. ChSP-1 leverages inspection data from both the current and immediately preceding lots to make acceptance decisions. By incorporating historical inspection outcomes, ChSP-1 enhances the discriminatory power of the sampling procedure leading to a steeper OC curve and a better evaluation of the lot quality (Govindaraju & Subramani 1993).

Despite its advantages, ChSP-1 offered insufficient protection to consumers as it does not decrease the chance of accepting poor quality lots compared to the zero-acceptance number plan. In response, the Modified Chain Sampling Plan (MChSP-1) was introduced, featuring more stringent acceptance conditions and more enhanced criteria for lot acceptance. These modifications improve the plan's ability to identify low-quality lots while still preserving favourable conditions for high-quality ones (Govindaraju & Lai 1998).

Sampling plans differ not only in methodology but also in their underlying acceptance criteria, which form the basis for determining lot acceptance or rejection. ChSP-1 employs five acceptance criteria, offering greater leniency and thus favouring producers. This makes it particularly appealing for industries where high rejection rates could result in substantial financial loss. Conversely, MChSP-1 adopts only three acceptance criteria, imposing stricter standards that prioritize consumer protection by lowering the chance of accepting bad lots. Recognizing the trade-off between these two approaches, the new chain sampling plans (NChSP-1) is developed in this study to offer a balanced alternative. NChSP-1 incorporates four acceptance criteria, situating it midway between ChSP-1 and MChSP-1 in terms of stringency. This design intends to mediate the conflicting interests of producers and consumers by offering a compromise that neither over penalizes producers nor excessively favours consumers. By doing so, it ensures that both parties are adequately protected.

Two inherent risks exist in acceptance sampling. They are the producer's risk, representing the probability of rejecting a good lot, and the consumer's risk, referring to the chance of accepting a bad lot (Chun & Rinks 1998). An effective sampling plan seeks to balance these risks to protect both stakeholders. As a result, three methods have been explored by the researchers to evaluate the sampling plan's performance which are minimizing consumer's risk (MCR), minimizing the sum of risks (MSR), and the minimum angle method (MAM).

MCR focuses solely on reducing consumer's risk by restricting the probability of lot acceptance must consistently below a pre-determined consumer's risk. Due to its appeal to consumers, MCR has been extensively studied by many researchers such as Alhyasat et al. (2025) and Jamaludin, Zain and Aziz (2017). Unfortunately, it often overlooking producer concerns. To address this limitation, the MSR method was introduced by Golub (1953) by minimizing the sum of both producer's and consumer's risk, ensuring a balanced consideration of both producer and consumer risks. MAM further improves upon this by targeting the closest possible alignment of the operating characteristic (OC) curve with an ideal curve, providing a more refined sampling plan design (Mohd Azri, Aziz & Zain 2022). Previous applications of MAM have demonstrated its effectiveness in balancing both risks such as Suresh and Vinitha (2014) for generalized two-plan systems, Jayalakshmi and Kaviyamani (2020) for repetitive group sampling plan, Mohd Azri, Aziz and Zain (2021) for new group chain sampling plan and Mohd Azri et al. (2025) for Bayesian new group chain sampling plans. These studies confirmed that MAM achieves optimal sample sizes satisfying both producers and consumers, unlike the MCR approach, which prioritizes only the consumer's perspective. Drawing on this framework, this study proposes NChSP-1, applying MAM with the generalized exponential (GE) distribution as the lifetime model. Table 1 shows the summary of the literature review.

## MATERIALS AND METHODS

The new chain sampling plan (NChSP-1) was systematically developed by incorporating the minimum angle method (MAM) through five sequential steps as outlined by the operational procedure in Table 2. Table 3 presents a comparison of the current lot sentencing result for ChSP-1, MChSP-1 and NChSP-1 when the preceding lot equals 2 based on their respective operational procedure. Table 3 illustrates that NChSP-1 provides a balanced alternative, employing four acceptance criteria and thus occupying an intermediate level of stringency between ChSP-1 with five acceptance criteria and MChSP-1 with only three acceptance criteria.

TABLE 1. Summary of the literature review

| References | Sampling Plan / Method | Contribution | Limitation |
|---|---|---|---|
| • Epstein (1954)<br>• Sadeghpour Gildeh, Baloui Jamkhaneh & Yari (2011)<br>• Kadir and Rahi (2023)<br>• Ahmed, Ali & Yousof (2024) | Single Sampling Plan (SSP) | The simplest and easiest to implement acceptance sampling plan | • Probability of lot acceptance declines sharply as the defective rate grows<br>• Requires a very large sample size to achieve the desired risk's levels |
| • Dodge (1955)<br>• Govindaraju & Subramani (1993) | Chain Sampling Plan (ChSP-1) | Offers better evaluation of lot quality and safeguards to the producers compared to SSP by considering preceding lots in current lot sentencing | • Too lenient towards producers (5 acceptance criteria)<br>• Less strict sampling plan raises the risk of committing type II error |
| • Govindaraju & Lai (1998) | Modified Chain Sampling Plan (MChSP-1) | Imposes more stringent conditions for lot acceptance than ChSP-1, resulting in improved protection for consumers | • Too strict towards producers (3 acceptance criteria)<br>• Stricter sampling plan raises the risk of committing type I error |
| • Jamaludin, Zain & Aziz (2017)<br>• Alhyasat et al. (2025) | Minimizing Consumer's Risk (MCR) | Naturally appealing to the consumers through capped consumer's risk | • Emphasis only on reducing consumer's risk and neglecting the producer's interest |
| • Golub (1953) | Minimum Sum of Risks (MSR) | Caters equally to both consumer's and producer's risk | • Does not aim for the closest possible match to an ideal OC Curve |
| • Suresh & Vinitha (2014)<br>• Mohd Azri, Nazrina & Zakiyah (2021)<br>• Mohd Azri, Nazrina & Zakiyah (2022)<br>• Mohd Azri et al. (2025) | Minimum Angle Method (MAM) | Caters equally to both consumer's and producer's risk and aims for the closest possible match to an ideal OC Curve | |

In the inspection of a sample of size $n$, each product unit is assessed independently, yielding only two possible outcomes: defective or non-defective. The probability that a product unit is defective in any inspection is denoted by $p$. This binary outcome structure makes the binomial model a natural choice for describing the probabilities of interest. Specifically, the probability of detecting exactly one defective unit, $P_1$ and the probability of finding no defective units, $P_0$ can be derived directly from the binomial distribution as $(1 - p)^n$ and $np\ (1 - p)^n$, respectively. Taking the preceding lots into account, the acceptance likelihoods under NChSP-1 involving $P_0$ and $P_1$ when number of preceding lots is 2 are shown in Table 4.

Based on the acceptance likelihood from each of acceptance criteria shown in Table 4, we can express the probability of lot acceptance, $L(p)$ for the current lot of NChSP-1 as in Equation (1):

$$L(p) = P_0^i[P_0 + (i+1)P_1] \tag{1}$$

Since $P_0$ and $P_1$ can be derived from the binomial distribution, the probability of lot acceptance is formulated as in Equation (2).

$$L(p) = (1-p)^{n(i+1)}\left[\frac{(i+1)np}{1-p} + 1\right] \tag{2}$$

The fraction defective quantifies the probability that a product unit fails to meet the required quality standard within the specified test termination time, $t_0$. It is equivalent to the probability that a product unit is defective in any inspection, as mentioned in the previous section and denoted as $p$. When the product lifetime is assumed to follow the generalized exponential (GE) distribution, as established by Mohd Azri, Aziz and Zain (2019), the value of $p$ can be expressed based on the cumulative distribution function (CDF) of the GE distribution as in Equation (3).

$$p = F(t_0;\lambda) = \left[1 - exp\left(-\frac{t_0}{\delta}\right)\right]^{\lambda} \tag{3}$$

where $\delta$ and $\lambda$ represent the scale parameter and shape parameter of GE distribution, respectively.

In the development of NChSP-1, the true mean life of the product is represented by $\mu$, while $\mu_0$ denotes the minimum acceptable mean life. The test termination time, $t_0$ is defined as $a\mu_0$ where $a$ is a constant. For the GE distribution, $\mu$ is equivalent to the scale parameter, $\delta$. To assess product quality, the mean ratio, $\frac{\mu}{\mu_0}$ is introduced. A higher mean ratio indicates superior quality performance. Accordingly, the fraction defective can be reformulated as a function of this mean ratio and can be expressed as in Equation (4).

$$p = F(a\mu_0;\lambda) = \left(1 - exp\left(\frac{-a}{\left(\frac{\mu}{\mu_0}\right)}\right)\right)^{\lambda} \tag{4}$$

By integrating Equations (2) and (4), the probability of lot acceptance for the NChSP-1 under the GE distribution can be obtained as in Equation (5).

$$L(p) = \left[1 - \left(1 - exp\left(\frac{-a}{\left(\frac{\mu}{\mu_0}\right)}\right)\right)^{\lambda}\right]^{n(i+1)}\left[\frac{(i+1)n\left(1-exp\left(\frac{-a}{\left(\frac{\mu}{\mu_0}\right)}\right)\right)^{\lambda}}{1-\left(1-exp\left(\frac{-a}{\left(\frac{\mu}{\mu_0}\right)}\right)\right)^{\lambda}} + 1\right] \tag{5}$$

In this study, the acceptable quality level (AQL) is defined as the highest proportion of defective units considered acceptable, whereas the lot tolerance percent defective (LTPD) represents the threshold at which a lot is deemed unacceptable and should be rejected with high probability. The associated risks include the producer's risk, $a$, which is the probability of rejecting a lot at the AQL and the consumer's risk, $\beta$, representing the probability of accepting a lot at the LTPD. To evaluate the NChSP-1, the minimum angle method (MAM) was employed to measure the proximity of its operating characteristic (OC) curve to an ideal reference. The following figure demonstrates the implementation of the minimum angle method following the approach of Sudamani Ramaswamy and Sutharani (2014).

Figure 1 depicts point A ($p_1$, $1 - a$) and point B ($p_2$, $\beta$), representing the acceptance probabilities at the AQL and LTPD, respectively. The minimum angle, $\theta$ is then derived by implying the tangent formula and can be simplified as in Equation (6).

$$\theta = arctan\left[\frac{p_2 - p_1}{L(p_1) - L(p_2)}\right] \tag{6}$$

The design parameters for the NChSP-1 are predetermined and include three key variables: the number of preceding lots, $i$, the mean ratio, $\frac{\mu}{\mu_0}$, and the fixed constant, $a$. The number of preceding lots is considered for four levels: 1, 2, 3, and 4. The mean ratio is tested at 2, 4, 6, and 8, while the fixed constant is set across a wider range of values, specifically 0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, and 2.00. The efficiency of the proposed plan is assessed through two primary performance parameters which are the optimal sample size, $n$ and the corresponding minimum angle, $\theta$. The optimal sample size is determined only when both probability of lot acceptance constraints are satisfied, namely (i) $L(p_1) \geq 1 - a$ and (ii) $L(p_2) \leq \beta$. If either condition is not met, the value of $n$ is excluded from consideration. In this study, a symmetric risk setting is implemented, where both producer's and consumer's risks are fixed at 0.10 for all cases. Subsequently, the

TABLE 2. Operational procedure for NChSP-1

| Step | Procedure |
|---|---|
| Step 1 | Draw a sample of size $n$ from the lot |
| Step 2 | Conduct the time truncated life test where the test is terminated at $t_0$ |
| Step 3 | Record the number of defective units, $d$ identified upon reaching $t_0$ |
| Step 4 | Assess the acceptance status of the current lot based on the following guideline: |
| | i. Case of $d = 0$ (current lot): |
| | If $i$ preceding lots have less than or equal to one defective unit, accept the current lot. Otherwise, reject the current lot. |
| | ii. Case of $d = 1$ (current lot): |
| | If no defective unit is identified in $i$ preceding lots, accept the current lot. Otherwise, reject the current lot. |
| | iii. Case of $d > 1$ (current lot): |
| | Reject the current lot |

TABLE 3. Current lot sentencing results for ChSP-1, MChSP-1 and NChSP-1 when $i = 2$

| Inspection outcome | | | Current lot sentencing | | |
|---|---|---|---|---|---|
| Preceding Lot 1 | Preceding Lot 2 | Current Lot | ChSP-1 | MChSP-1 | NChSP-1 |
| $d = 0$ | $d = 0$ | $d = 0$ | Accept | Accept | Accept |
| | | $d = 1$ | Accept | Reject | Accept |
| | $d = 1$ | $d = 0$ | Accept | Accept | Accept |
| | | $d = 1$ | Reject | Reject | Reject |
| $d = 1$ | $d = 0$ | $d = 0$ | Accept | Accept | Accept |
| | | $d = 1$ | Reject | Reject | Reject |
| | $d = 1$ | $d = 0$ | Accept | Reject | Reject |
| | | $d = 1$ | Reject | Reject | Reject |
| | | $d > 1$ | Reject | Reject | Reject |

TABLE 4. Acceptance likelihood under NChSP-1 when $i = 2$

| Inspection outcome | | | Acceptance likelihood |
|---|---|---|---|
| Preceding lot 1 | Preceding lot 2 | Current lot | |
| $d = 0$ | $d = 0$ | $d = 0$ | $(P_0)\,(P_0)\,(P_0)$ |
| | | $d = 1$ | $(P_0)\,(P_0)\,(P_1)$ |
| | $d = 1$ | $d = 0$ | $(P_0)\,(P_1)\,(P_0)$ |
| $d = 1$ | $d = 0$ | $d = 0$ | $(P_1)\,(P_0)\,(P_0)$ |

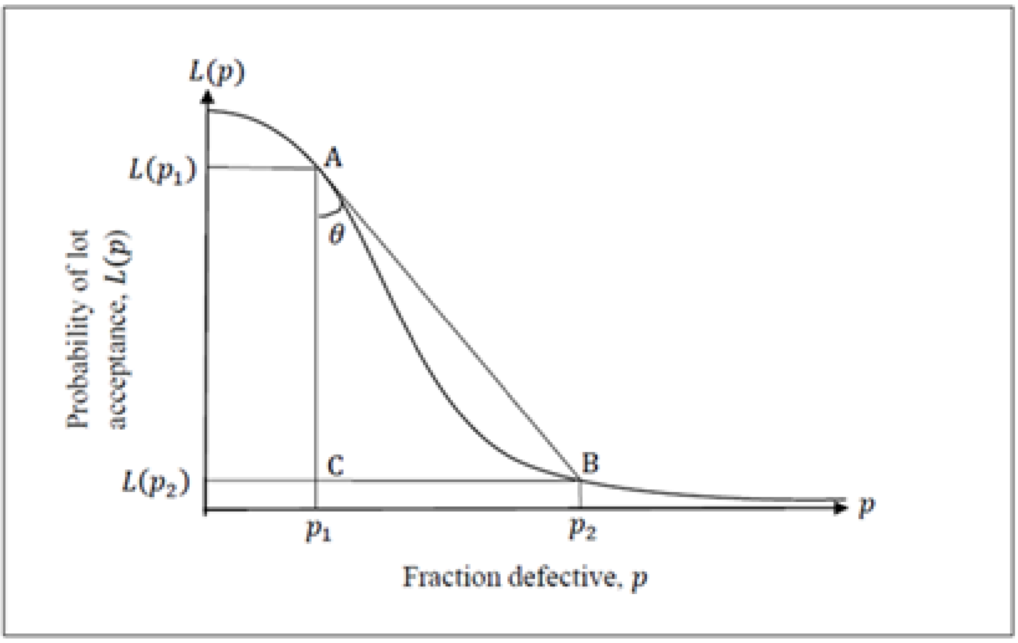


FIGURE 1. The minimum angle method (MAM) illustration

minimum angle, $\theta$ corresponding to each combination of design parameters is computed to evaluate the overall performance of the sampling plan. All computations in this study were carried out using Python programming. Python was used to check the acceptance probability conditions, test all combinations of design parameters, and obtain the optimal sample size and associated minimum angle.

## RESULTS AND DISCUSSIONS

The results of the NChSP-1 for lifetimes governed by the GE distribution were generated by following the operational procedure outlined in Table 2 and the predetermined design parameters. The optimal sample sizes and their associated minimum angles when $\lambda = 2$ and $\lambda = 3$ are summarised in Tables 5 and 6, respectively.

To clarify the interpretation of the results, consider a manufacturing company evaluating a production lot of smartphone batteries using the NChSP-1, assuming the product lifetimes follow the GE distribution. The consumer's and producer's risks are set at 0.10, while the mean ratio and number of preceding lots are fixed at 8 and 3, respectively. Based on these inputs, the sample size required for inspection must be determined. When the termination time is selected as one-half of the specified mean life, Table 5 indicates that a sample of twelve units is needed, yielding a minimum angle of 8.7382°. If the termination time is set at double the specified mean life, the optimal sample size decreases to two units, with an associated minimum angle of 36.4919°.

The patterns in Tables 5 and 6 highlight several important design characteristics. An increase in the fixed constant, $a$ leads to reductions in both the optimal sample size and the minimum angle. Similarly, larger numbers of preceding lots contribute to smaller required sample sizes. Conversely, an increase in the mean ratio, $\frac{\mu}{\mu_0}$ results in larger sample sizes. The (–) symbol indicates the combinations of design parameters for which no feasible optimal sample size exists, as the resulting plans fail to satisfy the required producer's and consumer's risk constraints of less than 0.10.

A performance comparison is carried out between the proposed NChSP-1 and the conventional ChSP-1. For this purpose, truncated life tests are regenerated using the operational procedure originally introduced by Dodge (1955) for ChSP-1. Each simulation run assumes that product lifetimes follow the GE distribution. The comparison concentrates on two key indicator which are optimal sample size and the associated minimum angle, under a fixed set of design parameters, specifically $(\lambda, \frac{\mu}{\mu_0}, i)$ = (2, 8, 3). The comparison results are presented in Figures 2 and 3.

As illustrated in Figure 2, the NChSP-1 requires a smaller optimal sample size compared to ChSP-1, implying improved cost efficiency due to fewer inspection units. Furthermore, Figure 3 shows that NChSP-1 attains a smaller MAM angle than ChSP-1, indicating a closer alignment with the ideal OC curve. Together, these results demonstrate that NChSP-1 not only reduces inspection effort but also more accurately approximates the desired operating characteristics of a sampling plan.

TABLE 5. Optimal sample size and minimum angle of NChSP-1 when λ = 2

| $\frac{\mu}{\mu_0}$ | $i$ | Parameter | $a$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
| 2 | 1 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| | 2 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| | 3 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| | 4 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| 4 | 1 | $n$ | 56 | 16 | 8 | 5 | - | - | 2 | - |
| | | $\theta°$ | 2.84350 | 8.93913 | 15.75748 | 21.94186 | - | - | 34.09136 | - |
| | 2 | $n$ | 38 | 11 | 6 | 3 | - | - | - | - |
| | | $\theta°$ | 2.84352 | 8.93710 | 15.78105 | 22.16875 | - | - | - | - |
| | 3 | $n$ | 28 | 8 | 4 | - | - | - | 1 | - |
| | | $\theta°$ | 2.84350 | 8.93913 | 15.75748 | - | - | - | 34.09136 | - |
| | 4 | $n$ | 23 | 7 | 3 | 2 | - | 1 | - | - |
| | | $\theta°$ | 2.84375 | 8.95013 | 15.81887 | 21.94186 | - | 31.12997 | - | - |
| 6 | 1 | $n$ | 70 | 20 | 10 | 7 | 5 | 4 | 3 | 3 |
| | | $\theta°$ | 2.79097 | 8.76294 | 15.46253 | 21.59464 | 26.64422 | 30.62346 | 33.51599 | 36.03540 |
| | 2 | $n$ | 47 | 14 | 7 | 4 | 3 | 2 | 2 | 2 |
| | | $\theta°$ | 2.79096 | 8.76326 | 15.45914 | 21.61552 | 26.63034 | 30.83203 | 33.51599 | 36.03540 |
| | 3 | $n$ | 35 | 10 | 5 | 3 | 2 | 2 | 2 | 1 |
| | | $\theta°$ | 2.79097 | 8.76294 | 15.46253 | 21.61552 | 26.78844 | 30.62346 | 34.22367 | 36.15718 |
| | 4 | $n$ | 28 | 8 | 4 | 3 | 2 | 2 | 1 | 1 |
| | | $\theta°$ | 2.79097 | 8.76294 | 15.46253 | 21.65168 | 26.64422 | 31.20742 | 33.74078 | 35.68891 |
| 8 | 1 | $n$ | 81 | 24 | 12 | 8 | 5 | 4 | 3 | 3 |
| | | $\theta°$ | 2.78414 | 8.73827 | 15.42191 | 21.55548 | 26.63959 | 30.61041 | 33.70170 | 35.94415 |
| | 2 | $n$ | 54 | 16 | 8 | 5 | 4 | 3 | 2 | 2 |
| | | $\theta°$ | 2.78414 | 8.73827 | 15.42191 | 21.54629 | 26.66103 | 30.63802 | 33.70170 | 35.94415 |
| | 3 | $n$ | 40 | 12 | 6 | 4 | 3 | 2 | 2 | 2 |
| | | $\theta°$ | 2.78416 | 8.73827 | 15.42191 | 21.55548 | 26.66103 | 30.61041 | 33.79088 | 36.49198 |
| | 4 | $n$ | 32 | 9 | 5 | 3 | 2 | 2 | 1 | 1 |
| | | $\theta°$ | 2.78416 | 8.73953 | 15.42573 | 21.54629 | 26.63959 | 30.74365 | 34.18057 | 35.98772 |
| 10 | 1 | $n$ | 89 | 26 | 13 | 8 | 6 | 5 | 4 | 3 |
| | | $\theta°$ | 2.78525 | 8.74150 | 15.43250 | 21.57439 | 26.67298 | 30.71556 | 33.79663 | 36.09705 |
| | 2 | $n$ | 59 | 17 | 9 | 6 | 4 | 3 | 3 | 2 |
| | | $\theta°$ | 2.78526 | 8.74164 | 15.43316 | 21.58157 | 26.67298 | 30.69267 | 33.88367 | 36.09705 |
| | 3 | $n$ | 45 | 13 | 7 | 4 | 3 | 2 | 2 | 2 |
| | | $\theta°$ | 2.78526 | 8.74150 | 15.43627 | 21.57439 | 26.67298 | 30.74032 | 33.79663 | 36.29900 |
| | 4 | $n$ | 36 | 10 | 5 | 3 | 2 | 2 | 2 | 1 |
| | | $\theta°$ | 2.78526 | 8.74219 | 15.43501 | 21.59052 | 26.74551 | 30.71556 | 34.00388 | 36.28182 |
| 12 | 1 | $n$ | 96 | 28 | 14 | 9 | 6 | 5 | 4 | 3 |
| | | $\theta°$ | 2.78762 | 8.74988 | 15.45074 | 21.60483 | 26.72974 | 30.76921 | 33.87797 | 36.25519 |
| | 2 | $n$ | 64 | 19 | 9 | 6 | 4 | 3 | 3 | 2 |
| | | $\theta°$ | 2.78762 | 8.74999 | 15.45226 | 21.60483 | 26.72974 | 30.78107 | 33.90944 | 36.25519 |
| | 3 | $n$ | 48 | 14 | 7 | 5 | 3 | 3 | 2 | 2 |
| | | $\theta°$ | 2.78762 | 8.74988 | 15.45074 | 21.61565 | 26.72974 | 30.82628 | 33.87797 | 36.30742 |
| | 4 | $n$ | 38 | 11 | 6 | 4 | 3 | 2 | 2 | 1 |
| | | $\theta°$ | 2.78764 | 8.74997 | 15.45264 | 21.61565 | 26.75453 | 30.76921 | 33.96855 | 36.50043 |

TABLE 6. Optimal sample size and minimum angle of NChSP-1 when λ = 3

| $\frac{\mu}{\mu_0}$ | $i$ | Parameter | $a$ = 0.25 | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| | 2 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| | 3 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| | 4 | $n$ | - | - | - | - | - | - | - | - |
| | | $\theta°$ | - | - | - | - | - | - | - | - |
| 4 | 1 | $n$ | 365 | 60 | 23 | 12 | 8 | 5 | 4 | 3 |
| | | $\theta°$ | 0.616607 | 3.466685 | 8.308221 | 14.07898 | 19.80127 | 24.84218 | 28.95912 | 32.21185 |
| | 2 | $n$ | 243 | 40 | 15 | 8 | 5 | 4 | 3 | 2 |
| | | $\theta°$ | 0.616607 | 3.466685 | 8.308158 | 14.07898 | 19.78793 | 24.90914 | 29.0602 | 32.21185 |
| | 3 | $n$ | 183 | 30 | 11 | 6 | 4 | 3 | 2 | 2 |
| | | $\theta°$ | 0.616607 | 3.466685 | 8.309329 | 14.07898 | 19.80127 | 24.90914 | 28.95912 | 32.61997 |
| | 4 | $n$ | 146 | 24 | 9 | 5 | 3 | 2 | 2 | 1 |
| | | $\theta°$ | 0.616607 | 3.466685 | 8.308158 | 14.08314 | 19.78793 | 24.84218 | 29.28668 | 32.72546 |
| 6 | 1 | $n$ | 470 | 77 | 29 | 16 | 10 | 7 | 5 | 4 |
| | | $\theta°$ | 0.617657 | 3.46925 | 8.308017 | 14.07634 | 19.7939 | 24.86537 | 29.06387 | 32.39584 |
| | 2 | $n$ | 313 | 52 | 20 | 10 | 7 | 5 | 3 | 3 |
| | | $\theta°$ | 0.617657 | 3.469252 | 8.30808 | 14.07652 | 19.79908 | 24.87864 | 29.10505 | 32.42816 |
| | 3 | $n$ | 235 | 39 | 15 | 8 | 5 | 3 | 3 | 2 |
| | | $\theta°$ | 0.617657 | 3.469252 | 8.30808 | 14.07634 | 19.7939 | 24.89549 | 29.11415 | 32.39584 |
| | 4 | $n$ | 188 | 31 | 12 | 6 | 4 | 3 | 2 | 2 |
| | | $\theta°$ | 0.617657 | 3.469247 | 8.30808 | 14.07652 | 19.7939 | 24.87864 | 29.06387 | 32.50667 |
| 8 | 1 | $n$ | 546 | 90 | 34 | 18 | 11 | 8 | 6 | 5 |
| | | $\theta°$ | 0.618785 | 3.476465 | 8.32774 | 14.11457 | 19.85813 | 24.96095 | 29.20043 | 32.58653 |
| | 2 | $n$ | 364 | 60 | 23 | 12 | 8 | 5 | 4 | 3 |
| | | $\theta°$ | 0.618785 | 3.476465 | 8.327726 | 14.11457 | 19.85843 | 24.96358 | 29.20043 | 32.58527 |
| | 3 | $n$ | 273 | 45 | 17 | 9 | 6 | 4 | 3 | 2 |
| | | $\theta°$ | 0.618785 | 3.476465 | 8.32774 | 14.11457 | 19.85843 | 24.96095 | 29.20043 | 32.62294 |
| | 4 | $n$ | 218 | 36 | 14 | 7 | 5 | 3 | 3 | 2 |
| | | $\theta°$ | 0.618785 | 3.476465 | 8.327755 | 14.11491 | 19.86039 | 24.96358 | 29.23349 | 32.58653 |
| 10 | 1 | $n$ | 606 | 101 | 38 | 20 | 13 | 9 | 7 | 5 |
| | | $\theta°$ | 0.619348 | 3.480395 | 8.339282 | 14.1383 | 19.89763 | 25.02106 | 29.28486 | 32.69545 |
| | 2 | $n$ | 404 | 67 | 26 | 14 | 9 | 6 | 5 | 4 |
| | | $\theta°$ | 0.619348 | 3.480394 | 8.339288 | 14.13838 | 19.89804 | 25.02106 | 29.28818 | 32.70075 |
| | 3 | $n$ | 303 | 50 | 19 | 10 | 6 | 5 | 3 | 3 |
| | | $\theta°$ | 0.619348 | 3.480395 | 8.339282 | 14.1383 | 19.89835 | 25.02383 | 29.28975 | 32.70075 |
| | 4 | $n$ | 242 | 40 | 15 | 8 | 5 | 4 | 3 | 2 |
| | | $\theta°$ | 0.619348 | 3.480395 | 8.339301 | 14.1383 | 19.89766 | 25.02383 | 29.28818 | 32.69545 |
| 12 | 1 | $n$ | 655 | 109 | 42 | 22 | 14 | 10 | 7 | 6 |
| | | $\theta°$ | 0.619637 | 3.482488 | 8.345639 | 14.15168 | 19.92057 | 25.05592 | 29.33308 | 32.75898 |
| | 2 | $n$ | 437 | 73 | 28 | 15 | 9 | 7 | 5 | 4 |
| | | $\theta°$ | 0.619637 | 3.482488 | 8.345639 | 14.15169 | 19.92064 | 25.05647 | 29.33291 | 32.75898 |
| | 3 | $n$ | 328 | 55 | 21 | 11 | 7 | 5 | 4 | 3 |
| | | $\theta°$ | 0.619637 | 3.482488 | 8.345639 | 14.15168 | 19.92057 | 25.05592 | 29.33408 | 32.75898 |
| | 4 | $n$ | 262 | 44 | 17 | 9 | 6 | 4 | 3 | 2 |
| | | $\theta°$ | 0.619637 | 3.482488 | 8.345644 | 14.15169 | 19.92095 | 25.05592 | 29.33291 | 32.76512 |

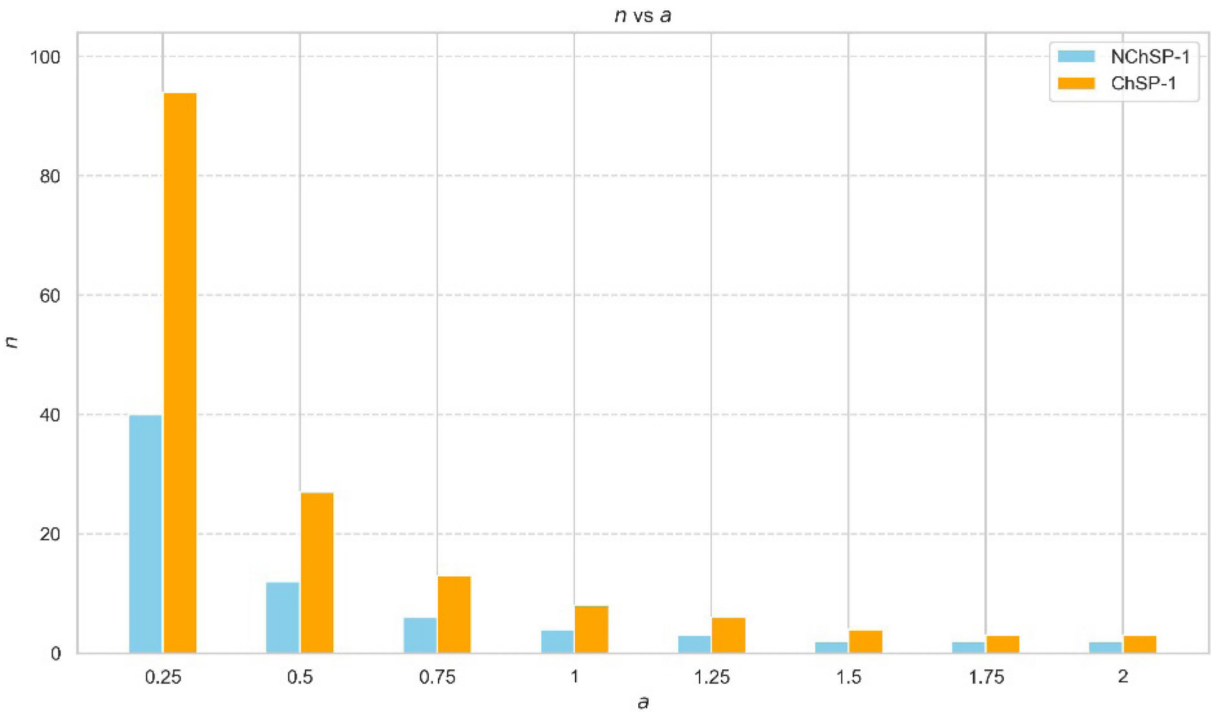


FIGURE 2. Optimal sample size comparison between ChSP-1 and NChSP-1 using MAM with combination of design parameters $(\lambda, \frac{\mu}{\mu_0}, i) = (2,8,3)$

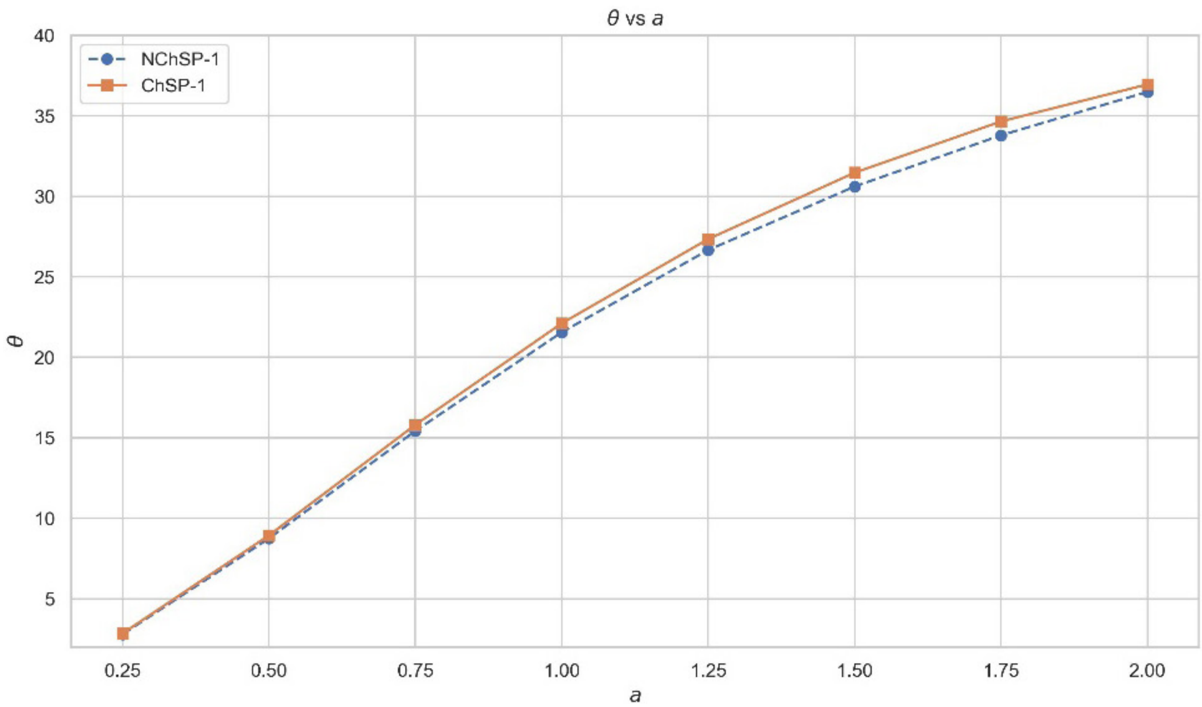


FIGURE 3. Minimum angle comparison between ChSP-1 and NChSP-1 using MAM with combination of design parameters $(\lambda, \frac{\mu}{\mu_0}, i) = (2,8,3)$

## CONCLUSIONS

The NChSP-1 was developed using the GE distribution and evaluated with the minimum angle method, employing four acceptance criteria to balance the leniency of ChSP-1 and the stringency of MChSP-1. Performance assessment based on sample size and associated minimum angle, indicates that NChSP-1 is more cost-efficient than ChSP-1 and achieves a closer fit to the ideal OC curve across varying fixed constant levels. These results highlight NChSP-1 as a practical and effective option for industries engaged in product life testing.

The proposed NChSP-1 has several limitations. First, the plan is developed under the assumption that product lifetimes follow the GE distribution, which may not adequately represent all practical situations where other distributions such as Weibull or lognormal are more appropriate. Second, the design of the sampling plan is based on fixed producer's and consumer's risk levels, which may not reflect varying industrial requirements. Therefore, the applicability and performance of the proposed plan may be limited under different distributional assumptions or risk settings in practical applications.

Future research may extend the NChSP-1 in several directions. One possible extension is the development of similar sampling plans under alternative lifetime distributions such as Weibull, lognormal or gamma distributions. In addition, future studies may investigate the performance of the proposed plan under different risk configurations, such as asymmetric producer's and consumer's risks. Empirical validation using real industrial datasets may also provide further insight into the practical applicability of the proposed sampling plan.

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