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Credible Delta Gamma (Theta) Normal Value at Risk for Assessing European Call Option Risk

(Nilai Normal Delta-Gamma (Theta) Berisiko Boleh Percaya untuk Menilai Risiko Pilihan Panggilan Eropah)

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ABSTRACT

The current research introduces a novel risk metric called credible delta-gamma (theta)-normal Value-at-Risk (CredDGTN VaR) for the purpose of the option risk assessment. CredDGTN VaR represents an extension of the credible Value-at-Risk (CredVaR) framework, whereby risk assessment is conducted through the integration of CredVaR with delta-gamma(theta)-normal VaR. The present study introduces a novel approach that is deemed suitable for evaluating the risk of a portfolio of European call options. The proposed method takes into account the nonlinear interdependence of the market risk factors determining the value of a European call option, according to the Formula of Black-Scholes. The present methodology is employed to assess simulated financial data that portrays the return of multiple assets throughout ten investment periods. The novel approach is additionally employed to assess the level of risk associated with a portfolio comprised of actively traded stock options. According to Kupiec's backtesting, CredDGTN's efficacy in gauging the risk of an option portfolio is noteworthy, as it accurately measures the risk at 80%, 90%, and 95% confidence levels, even in cases where the profit/loss (P/L) exhibits non-normal distribution. Furthermore, the performance of CredDGTN VaR empirically outperforms credible delta-normal VaR (CredDN VaR) and credible delta-gamma-normal VaR (CredDGN VaR) in similar cases. Moreover, CredDN VaR, CredDGN VaR, and CredDGTN VaR will provide equal VaR when delta and gamma are zero.

Keywords: Greek; mixed-assets; portfolio

ABSTRAK

Penyelidikan ini memperkenalkan metriks risiko baharu yang dipanggil nilai normal delta-gamma (theta) berisiko boleh percaya (CredDGTN VaR) untuk tujuan penilaian risiko pilihan. CredDGTN VaR mewakili lanjutan daripada rangka kerja Nilai Berisiko (CredVaR) boleh percaya yang mana penilaian risiko dijalankan melalui penyepaduan CredVaR dengan delta-gamma(theta)-normal VaR. Kajian ini memperkenalkan pendekatan baharu yang dianggap sesuai untuk menilai risiko portfolio pilihan panggilan Eropah. Kaedah yang dicadangkan mengambil kira kebergantungan tidak linear faktor risiko pasaran yang menentukan nilai pilihan panggilan Eropah, menurut Formula Black-Scholes. Metodologi sedia ada digunakan untuk menilai simulasi data kewangan yang menggambarkan pulangan berbilang aset sepanjang sepuluh tempoh pelaburan. Pendekatan baharu ini digunakan untuk menilai tahap risiko yang berkaitan dengan portfolio yang terdiri daripada pilihan saham yang didagangkan secara aktif. Menurut pengiraan ke belakang Kupiec, keberkesanan CredDGTN dalam mengukur risiko portfolio pilihan patut diberi perhatian, kerana ia mengukur risiko dengan tepat pada tahap keyakinan 80%, 90% dan 95%, walaupun dalam kes keuntungan/kerugian (P/L) menunjukkan taburan bukan normal. Tambahan pula, prestasi CredDGTN VaR secara empirik mengatasi VaR delta-normal boleh percaya (CredDN VaR) dan VaR delta-gamma-normal boleh percaya (CredDGN VaR) dalam kes yang serupa. Selain itu, CredDN VaR, CredDGN VaR dan CredDGTN VaR akan memberikan VaR yang sama apabila delta dan gamma adalah sifar.

Kata kunci: Aset gabungan; Greek; portfolio

INTRODUCTION

VaR has quickly become a standard quantitative benchmark for assessing the risk exposure of a portfolio. VaR offers an upper bound on the potential loss of a portfolio at a certain time horizon and confidence level, with a greater confidence level indicating a lower possibility that this loss would be exceeded (Zhao et al. 2015).

In order to compute the Value at Risk (VaR) of an option, it is imperative to possess information regarding the profit or loss (P/L) of options that are not currently accessible on the capital market, as noted by Zymler, Kuhn and Rustem (2013). In contrast to stock prices, which exhibit a linear relationship with market risk factors, the valuation of an option demonstrates a nonlinear relationship with market risk factors, as stated by Chen and Yu (2013). The direct calculation of VaR from market risk factors in portfolios containing options is hindered by nonlinearity, as stated by Ortiz-Gracia and Oosterlee (2014).

Many approaches for calculating the risk of options, such as delta-normal VaR (DN VaR) and delta-gamma-normal VaR (DGN VaR), have been created. In contrast, the second-order Taylor Polynomial is used by DGN VaR employing a quadratic approximation of the asset value (Date & Bustreo 2016) to approximate the P/L of the underlying assets (Sulistianingsih, Rosadi & Abdurakhman 2019). Numerous researchers, such as Britten-Jones & Schaefer (1999), Castellacci and Siclari (2003), Cui et al. (2013), Date and Bustreo (2016), Feuerverger and Wong (2000), Mina and Ulmer (1999), Ortiz-Gracia and Oosterlee (2014), and Wang et al. (2017), have investigated and utilized both methodologies.

Bühlmann's credibility theory, as presented in the work of Bühlmann's (Bühlmann 1969), integrates the notions of individual risk and group risk to evaluate the credibility of insurance premiums effectively. The application of credibility theory has been observed in the domains of insurance and finance. Credibility theory has been extensively utilized in various studies within the field of insurance. One individual was subjected to execution by Diao and Weng (2019). In the field of financial research, the application of credibility theory has been utilized to construct a measure of risk for a given portfolio, with consideration given to both fuzzy and nonfuzzy concepts. This approach has been explored by various scholars, including Chen, Liu and Chen (2006), Georgescu and Kinnunen (2013), Liu, Chen and Liu (2018), Pitselis (2013), Vercher and Bermúdez (2015), and Wang, Chen and Liu (2016). However, the studies mentioned earlier failed to investigate derivative instruments, including options. The risk investigation of derivative instruments, namely stock options, was conducted by Sulistianingsih, Rosadi and Abdurakhman (2023, 2021).

Credibility theory was utilized to show how quantiles can be integrated into Bühlmann's classical credibility model in Pitselis (2013). Pitselis introduced CredVaR, a new risk metric that blends VaR with credibility methodology. Pitselis (2016) suggested that CredVaR is considered more informative than VaR because it can capture the risk of an individual insurer's contract (or asset return). This measure can also represent the portfolio risk of similar (but not identical) contracts (or the returns of a portfolio of similar assets) that are pooled to share risk. Many finance scholars have applied credibility theory, but not to option risk. Thus, the risk measures have ignored the nonlinear dependency between derivative asset values like options and their risk factors.

The significance of options in controlling market risk has increased the importance of risk measurement tools for option portfolios (Topaloglou, Vladimirou & Zenios 2011). The tools for measuring the risk of options based on credibility theory were developed previously, such as Credible Delta Gamma Normal VaR (CredDGN VaR) initiated by Sulistianingsih, Rosadi and Abdurakhman (2021) and Credible Delta Normal VaR (CredDN VaR) created by Sulistianingsih, Rosadi and Abdurakhman (2023). Both risk measures have not yet considered the sensitivity of option price relative to the change of time (theta). For these reasons, we propose a technique to evaluate the risk of a European call option. This approach integrates delta-gamma (theta)-normal (DGTN) VaR with CredVaR. The method is later called credible delta-gamma (theta) normal Value-at-Risk (CredDGTN VaR). CredDGTN VaR employs more option risk sensitivity measures rather than CredDN VaR and CredDGN VaR, so it is expected to be more powerful when the method is implemented in measuring a call option risk.

This paper is structured in the following manner. Next section outlines the theoretical framework employed in the development of the proposed VaR model. After that the paper presents a novel risk-measurement model, CredDGTN VaR. The proposed model is applied to option profit/loss data generated under the assumption of normality in the following section. Subsequent section employs the model to evaluate the risk associated with stock options that are traded in the financial market. Ultimately, last section presents a conclusive analysis and comments for future research.

VALUE AT RISK UTILIZING DELTA-GAMMA (THETA)-NORMAL APPROXIMATION

The option is a type of financial derivative that enables investors to manage risk effectively (Yang, Ma & Liang 2018). The value of a European call option at time t can be represented as a multivariable function, denoted as $C_t = f(S_t, K, r, t, \sigma)$, and can be expressed as Equation (1).

$$C_t = S_t N(d_1) - Ke^{-r(T-t)}N(d_2) \quad (1)$$

The variables denoted as S_t, K, r, t, and σ represent the asset price at time t, the exercise price, the risk-free interest rate, the expiration date, and the volatility of the asset, respectively as stated by Kananthai and Suksern (2016). Meanwhile, Ammann and Reich (2001) stated that

$$d_1 = \frac{ln(S_t/K)+(T-t)(r+\sigma^2/2)}{\sigma\sqrt{(T-t)}} \text{ and } d_2 = d_1 - \sigma\sqrt{(T-t)}.$$

Several indicators are used to quantify option risk based on Equation (1). The Greeks are employed in the formulation of the approach used to assess these risks. Greeks which are used to develop credible delta-gamma (theta) normal are delta (δ), gamma (γ), and theta (Θ). Delta measures the change in the option value/price in response to a change in the underlying asset price. Gamma measures the risk of a change in a delta option due to an asset-value change. Meanwhile, theta quantifies the sensitivity of the option price relative to the change of time. Delta, Gamma, and Theta for the European call option can be expressed as in Equations (2), (3), and (4).

$$\delta = N(d_1) \quad (2)$$

$$\gamma = N'(d_1)\frac{1}{S_t\sigma\sqrt{(T-t)}}. \quad (3)$$

$$\Theta = \frac{S_0\sigma}{2\tau^{\left(\frac{1}{2}\right)}}N'(d_1) - rKe^{-r(T-t)}N(d_2). \quad (4)$$

In the following analysis, we employ the delta δ,γ, and Θ Greeks, which are represented by Equations (2), (3), and (4) correspondingly, to construct the nonlinear credible VaR using the delta-gamma (theta)-normal approximation (DGTNA).

Under the assumption that the option value, denoted as C_t in Equation (1), is solely affected by its underlying asset and that K, r, t, and σ remain constant, the option value can be represented by a function of the underlying asset value, denoted as S_t, as shown in Equation (5):

$$C_t \approx f(S_t) \quad (5)$$

In order to assess the risk of an option through the utilization of VaR via a DGTNA, certain assumptions are necessary. According to Sulistianingsih, Rosadi and Abdurakhman (2019), it is assumed that there exists a nonlinear relationship between the P/L of a stock and a change in the option price. The second is that the P/L of the stock that underlies the option conforms to a normal distribution characterized by a volatility of $\sigma_{\Delta S_{t+\Delta t}}$ and a mean zero.

The present study focuses on the estimation of the nonlinear VaR associated with options through the utilization of the DGTNA. This approach takes into account the changing value of the underlying asset, specifically stocks. Similar to delta gamma-normal approximation (DGNA), the DGTNA also necessitates a second-order Taylor Polynomial to approximate P/L option value around S_t:

$$\Delta C_t \approx f(S_t + \Delta S_t) - f(S_t) \approx \left(\frac{\partial C_t}{\partial S_t}\right)\Delta S_t + \frac{1}{2}\left(\frac{\partial^2 C_t}{\partial^2 S_t^2}\right)(\Delta S_t)^2 + \Theta\Delta t, \quad (6)$$

where $\left(\frac{\partial C_t}{\partial S_t}\right)$ and $\left(\frac{\partial^2 C_t}{\partial \ S_t^2}\right)$, and Θ are suggested, respectively, in Equations (2), (3), and (4) as the Delta Greek (δ), gamma Greek (γ), and theta Greek (Θ) of an option, so Equation (6) can be expressed by Equation (7).

$$\Delta C_{t+\Delta t} \approx \delta\Delta S_t + \frac{\gamma}{2}(\Delta S_t)^2 + \Theta\Delta t. \quad (7)$$

Based on Equation (7), mean and variance of the stock option profit/loss for DGTNA VaR are consecutively $\Theta\Delta t$ and $\delta^2\sigma_{\Delta S_t}^2 + \frac{\gamma^2}{4}\sigma_{\Delta S_t}^4$. Then, VaR using DGTNA, called DGTN VaR, can be derived by replacing the mean and the variance in the formula of VaR based on Normal Distribution with $\Theta\Delta t$ and $\delta^2\sigma_{\Delta S_t}^2 + \frac{\gamma^2}{4}\sigma_{\Delta S_t}^4$. So DGTN VaR can be expressed in Equation (8) (Sulistianingsih, Rosadi & Abdurakhman 2023)

$$VaR_{DGTN} \approx Z_{1-\alpha}\sqrt{\delta^2\sigma_{\Delta S_t}^2 + \frac{\gamma^2}{4}\sigma_{\Delta S_t}^4} - (\Theta\Delta t), \quad (8)$$

where $Z_{1-\alpha}$ is a standard normal variate so that $1 - \alpha$ of mass density probability is located on the left side and α of the mass density probability is located on the right side.

CREDIBLE DELTA GAMMA (THETA) NORMAL VALUE AT RISK FOR

MEASURING THE OPTION RISK

This section provides an in-depth review of credibility theory and credible value at risk (VaR). The theory of credibility was formulated by Bühlmann (1969) and Bühlmann & Straub (1970). The Bühlmann concept of credibility is employed in credibility theory in ascertaining the insurance product premium. A claim made by the policyholder can be regarded as a risk that necessitates forethought and consideration by the insurance appraiser. In an attempt to proactively address a potentially unfavorable scenario wherein the number of claims exceeds the available funds possessed by the assessor, Bühlmann (1969) developed the Bühlmann Credibility as a valuable tool that can be employed to make estimations regarding claims for the upcoming period.

Pitselis (2016) introduced a novel risk measure that merges the credibility theory concept, originally suggested by Bühlmann (1969), with the widely used financial and insurance risk measurement tool, VaR. The new risk measure tool is known as credible VaR. In this paper, we utilize the concept constructed by Pitselis (2016) and Sulistianingsih, Rosadi and Abdurakhman (2023, 2021) to propose a novel measurement method for stock-option risk. This new risk-measurement methodology, inspired by the research of Pitselis (2016), is developed by combining credible VaR with DGTN VaR, which uses more option sensitivity measure in approximating risk.

Theorem 1 A random vector ϖ_j^* representing Delta Gamma (Theta) Normal VaR (DGTN VaR), $\eta_{i,j}^*$, of j asset in period i = 1,2,...,n where j = 1,2,...,m is expressed by

$$\varpi_j^{*'} = (\eta_{1,j}^* \dots, \eta_{n,j}^*). \tag{9}$$

Then, $\eta_{1,j}^* \dots, \eta_{n,j}^*$ is assumed identically distributed with *mean* $E(\eta_{i,j}^*) = \mu^*$ and variance $Var(\eta_{i,j}^*) = \sigma^2$. Next, the risk of the stock-option profit/loss in a portfolio is represented by a random variable ϑ^*, which is unknown distributed. A random variable $\eta_{1,j}^* \dots, \eta_{n,j}^*$ assumed conditional iid for a fixed ϑ^* where:

$$E(\eta_{i,j}^* \mid \vartheta^* = v^*) = \mu^*(v^*) \tag{10}$$

and

$$Var(\eta_{i,j}^* \mid \vartheta^* = v^*) = \tau^*(v^*) \tag{11}$$

for $i = 1,2,\dots,n$.

Then, by adopting Cred VaR based on Bühlmann credibility in the concept of risk measurement, the nonlinear VaR of j^{th} asset in the next period later called Credible Delta Gamma (Theta) Normal VaR (CredDGTN VaR), symbolized by Ψ_{DGTN} will be estimated.

Based on the assumption utilized in Cred VaR, a linear estimator of CredDGTN VaR for j^{th} asset developing a portfolio can be expressed by Equation (12).

$$\Psi_{DGTN}(v^*) = E[\eta_{i,j}^*]Z_{DG} + (1 - Z_{DG})\mu^*(v^*), \tag{12}$$

where $E(\eta_{i,j}^*)$ is mean of DGTN VaR for j^{th} asset during the observed period; u^* (v^*) is mean of DGTN portfolio VaR and Z_{DGTN} is a risk factor of CredDGTN VaR defined in Equation (13).

$$Z_{DGTN} = \frac{nVar(\mu^*(v^*))}{E[\tau^*(v^*)] + nVar[\mu^*(v^*)]}, \tag{13}$$

where $Var[\mu^*(v^*)]$ is a variance of DGTN VaR mean and $E[\tau^*(v^*)]$ is mean of DGTN VaR variance.

Proof Define an estimator of $E[\eta_{n+1,j}^* | v^*]$ as $g_0^* + \sum_{i=1}^n g_i^* \eta_{i,j}^*$, where g_0^* and g_i^* is specified so that it can minimize Equation (14).

$$R = E[\mu^*(v^*) - g_0^* - \textstyle\sum_{i=1}^n g_i^* \eta_{i,j}^*]^2. \tag{14}$$

After that, take the derivative of Equation (14) which is subsequently relative to g_0^* and g_k^*

Derivatife of R relative to g_0^*, with $\frac{\partial R}{\partial g_0^*} = 0$ is $E[\mu^*(v^*)] = g_0^* + \sum_{i=1}^n g_i^* E[\eta_{i,j}^*]$. It can be written that

$$E[\mu^*(v^*)] = g_0^* + \textstyle\sum_{i=1}^n g_i^* E[\eta_{i,j}^*]. \tag{15}$$

$$\mu^*(v^*) = g_0^* + \sum_{i=1}^n g_i^* \mu^*(v^*).$$

$$g_0^* = \mu^*(v^*)\left[1 - \sum_{i=1}^n g_i^*\right].$$

Then, because $\frac{\partial^2 R}{\partial g_0^{*2}} = 2$ so $R''(\mu^*(v^*) - \mu^*(v^*)\sum_{i=1}^n g_i^*) > 0$.

Therefore, $R(\mu^*(v^*) - \mu^*(v^*)\sum_{i=1}^n g_i^*)$ is a local minimum of R.

Next, derivative R relative to g_k^*, with $\frac{\partial R}{\partial g_k^*} = 0$ is

$$E[\mu(\theta)\eta_{k,j}^*] = g_0^* E[\eta_{k,j}^*] + \textstyle\sum_{i=1}^n g_i^* E[\eta_{i,j}^* \eta_{k,j}^*]. \tag{16}$$

Then, Equation (15) is multiplied by $E[\eta_{k,j}^*]$ and subtracted from Equation (16), so that it can be obtained as follows,

$$Cov[\mu^*(v^*), \eta_{k,j}^*] = \textstyle\sum_{i=1}^n g_i^* Cov[\eta_{i,j}^*, \eta_{k,j}^*]. \tag{17}$$

Consider that $i \neq k$

$$Cov(\eta^*_{i,j}, \eta^*_{k,j}) = Cov[E(\eta^*_{i,j} | v^*), E(\eta^*_{k,j} | v^*)] + E[Cov(\eta^*_{i,j}, \eta^*_{k,j} | v^*)]$$

$$= Cov[\mu^*(v^*), \mu^*(v^*)] + 0$$

$$= Var(\mu^*(v^*)) .$$

Next,

$$Cov[\mu^*(v^*), \eta^*_{k,j}] = E[\mu^*(v^*)\eta^*_{k,j}] - E[\mu^*(v^*)]E[\eta^*_{k,j}] .$$

$$= Var[\mu^*(v^*)]$$

So, $Cov[\mu^*(v^*), \eta_{k,j}] = \sum_{i=1}^{n} g_i Cov\left(\eta^*_{i,j}, \eta^*_{k,j}\right)$,

and $g_i^* = \frac{Var[\mu^*(v^*)]}{E[\tau^*(v^*)]+nVar[\mu^*(v^*)]}$ (18)

Next, for $i = k$, $\frac{\partial^2 R}{\partial \ g_k^{*2}} = 2E\left(\eta^{*2}_{k,j}\right)$. Because $E\left(\eta^{*2}_{k,j}\right)$ is positive, it can be concluded that $R(g_i^*)$, where $g_i^* = \frac{Var[\mu^*(v^*)]}{E[\tau^*(v^*)]+nVar[\mu^*(v^*)]}$ is a local minimum of R.

It can be obtained

$$g_0^* + \sum_{i=1}^{n} g_i^* \eta^*_{i,j}$$

$$= \left[1 - \sum_{i=1}^{n} g_i^*\right]\mu^*(v^*)$$

$$+ \sum_{i=1}^{n} \frac{Var(\mu^*(v^*))}{E[\tau^*(v^*)] + nVar(\mu^*(v^*))}\eta^*_{i,j}$$

$$= \left[1 - \sum_{i=1}^{n} \frac{Var(\mu^*(v^*))}{E[\tau^*(v^*)] + nVar(\mu^*(v^*))}\right]\mu^*(v^*)$$

$$+ \frac{n}{n}\sum_{i=1}^{n} \frac{Var(\mu^*(v^*))}{E[\tau^*(v^*)] + nVar(\mu^*(v^*))}\eta^*_{i,j}$$

$$= \left[1 - \sum_{i=1}^{n} \frac{Var(\mu^*(v^*))}{E[\tau^*(v^*)] + nVar(\mu^*(v^*))}\right]\mu^*(v^*)$$

$$+ \frac{nVar(\mu^*(v^*))}{E[\tau^*(v^*)] + nVar(\mu^*(v^*))}\sum_{i=1}^{n}\frac{1}{n}\eta^*_{i,j}$$

$$= \left\{1 - \frac{nVar(\mu^*(v^*))}{E[\tau^*(v^*) + nVar[\mu^*(v^*)]]}\right\}[\mu^*(v^*)]$$

$$+ \frac{nVar[\mu^*(v^*)]}{E[\tau^*(v^*)] + nVar[\mu^*(v^*)]}E[\eta^*_j]$$

$$= (1 - Z_{DG})\mu^*(v^*) + Z_{DG}E[\eta^*_j] ,$$

resulting in Z_{DGTN} as given in Equation (13).

Certain information is necessary to perform a risk estimation of an asset utilizing CredDGTN VaR. This information includes $E[\eta^*_j]$, $\mu^*(v)$, $E[Var(\eta^*_{i,j}|v^*)] = E[\tau^*(v^*)]$, and $Var\ E[\eta^*_{i,j}|v^*] = Var[\mu^*(v^*)]$. Given the aforementioned assumptions regarding the unknown distribution of a random variable ϑ^*, the mean sample formula is utilized to estimate $\mu^*(v^*), E[\tau^*(v^*)]$, and $Var[\mu^*(v^*)]$ based on the available data.

The estimator of $E[\eta^*_j]$, $\mu^*(v), E[\tau^*(v^*)]$, and $Var[\mu^*(v^*)]$, which are unbiased, are provided subsequently by Equations (19), (20), (21), and (22).

$$\widehat{E[\eta^*_j]} = \frac{1}{n}\sum_{i=1}^{n}\eta^*_{i,j} . \quad (19)$$

$$\widehat{\mu^*(v^*)} = \frac{1}{mn}\sum_{j=1}^{m}\sum_{i=1}^{n}\eta^*_{i,j} . \quad (20)$$

$$E[\widehat{\tau^*(v^*)}] = \frac{1}{m(n-1)}\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\eta^*_{i,j} - \bar{\eta}^*_j\right)^2 . \quad (21)$$

$$Var[\widehat{\mu^*(v^*)}] = \frac{1}{m-1}\sum_{j=1}^{m}\left(\bar{\eta}^*_j - E[\widehat{\tau^*(v^*)}]\right)^2 - \frac{E[\widehat{\tau^*(v^*)}]}{n} \quad (22)$$

The recursive process that includes several steps in the derivation of CredDGTN VaR and its credible factors is not presented here for brevity. The recursive process is similar to the recursive process developed by Sulistianingsih, Rosadi and Abdurakhman (2023, 2021). We only substitute the DN VaR and DGN VaR with DGTN VaR, which is considered to complete the previous methods with more option-greek to quantify the risk in its approximation.

APPLICATION OF CREDIBLE DELTA-GAMMA (THETA)-NORMAL

VALUE-AT-RISK USING DATA SIMULATION

Herein, we evaluate four P/L portfolios utilizing the theory discussed in the previous section. Portfolio I consists of two options, namely option A and option B; Portfolio II consists of two assets, namely option A, option B, and option C; Portfolio III consists of three assets, namely option A, option B, option C, and option D; meanwhile Portfolio IV consists of five assets namely option A, B, C, D, and E. The P/L data were generated using the same simulation procedure for obtaining ten periods of DGTN VaR for each option. These ten-period data were the same as the data analyzed in Sulistianingsih et al. (2023). The data was considered as ten-year data fabricated to meet the assumptions for the proposed VaR approximation.

Using the similar assumption in credible delta-gamma-normal (CredDGN) VaR developed by Sulistianingsih, Rosadi and Abdurakhman (2021), we determine the zero mean of the option's P/L, the standard deviation of option's P/L, the specified option price at time t (S_t), time to maturity (Δt), and exercise price (K) of each analyzed option, as listed in Table 1. The interest rate of risk-free assets (r) was specified as 0.0175 for each evaluated option.

We employed a recursive procedure comprised of Stages 1 through 10 provided in Sulistianingsih, Rosadi and Abdurakhman (2021) by substituting DGN VaR with DGTN VaR to estimate CredDGTN VaR for each asset with a given confidence level (cl) for each period (period one to period ten). The DGTN VaR for assets A, B, C, and D with a 95% confidence level for each period (period one to period ten) is calculated using Equation (8) and is provided in Table 2. Next, the DGTN VaR of four assets for confidence levels of 99%, 90%, and 80% can be calculated using a similar technique.

It can be deduced from Table 2 that the estimated potential losses for each asset A, B, C, and D with an 80 percent level of confidence in the first period over a day holding period were, respectively, 0.424, 0.852, 1.267, and 1.690 dollars relative to the asset's price on the previous day. It is simple to calculate and interpret the DGTN VaR for asset A, asset B, asset C, and asset D for 99%, 95%, and 90% confidence levels analogously. The quantification of DGTN VaR for the three specified levels of confidence is also provided in Table 2. Meanwhile, the estimated parameters utilized to calculate CredDGTN VaR for each asset constructing the portfolio, namely μ^*, τ^*, α^* and Z_{DGTN} at the determined confidence levels, are calculated and listed in Table 3. Table 3 demonstrates that $\widehat{Z_{DGTN}}$ for the examined portfolios is closer to one. Therefore, the CredDGTN VaR of each option constructing the three option portfolios tends to be equal to the estimated mean of the corresponding DGTN VaR for each asset at the specified confidence levels. The parameters needed in the CredDGTN VaR computation for each asset in the portfolio are quantified based on Equations (20), (21), and (22). Table 3 provides a summary of the estimators for $\mu^*, \tau^*,$ and $\alpha^*,$ at Portfolio I, Portfolio II, and Portfolio III for 99%, 95%, 90%, and 80% confidence levels. Based on the results, it can be noted that the mean of the maximum potential loss at 80%, 90%, 95%, and 99% confidence levels (cl) for each asset in Portfolio I were, successively, 0.638 dollars, 0.971 dollars, 1.247 dollars, and 1.763 dollars. Furthermore, the estimated mean of DGTN VaR variance at 80%, 90%, 95%, and 99% confidence levels for each asset in Portfolio I was 0.001, 0.002, 0.003, and 0.006, successively, while the variances of the DGTN VaR mean for each asset at Portfolio I at 80%, 90%, 95%, and 99% confidence levels were 0.092, 0.213, 0.351, and 0.701, respectively.

The results indicate that for Portfolio I, the maximum potential loss at various confidence levels (80%, 90%, 95%, and 99%) for each asset had an average value of 0.638, 0.971, 1.247, and 1.763 dollars. In addition, the mean of DGTN VaR variance at 80%, 90%, 95%, and 99% confidence levels for each asset in Portfolio I was 0.001, 0.002, 0.003, and 0.006, respectively, while the variances of the DGTN VaR mean for each asset at Portfolio I at 80%, 90%, 95%, and 99% confidence levels were 0.092, 0.213, 0.351, and 0.701, respectively. Specifically, the results report the mean and variance of DGTN VaR at different confidence levels for each asset in the portfolio. The mean values of DGTN VaR variance increase as the confidence level increases, with the highest mean value at the 99% confidence level. The variances of the DGTN VaR mean also increase as the confidence level increases, with the highest variance at the 99% confidence level.

Subsequently, based on the data presented in the aforementioned tables, the risk factors of CredDGTN VaR for Portfolio I, Portfolio II, Portfolio III, Portfolio IV, and Portfolio V were estimated using Equations (12) and (13), yielding values of 0.999, 0.999, 0.999, and 0.999, respectively. Table 4 presents the Cred-DGTN values for the four assets in Portfolios I, II, and III at the 80%, 90%, 95%, and 99% confidence levels.

When a risk measurement technique satisfies the necessary properties of theoretical statistics, it is considered to be well-specified, as noted by Karimalis and Nomikos (2018). The determination of whether a method satisfies this criterion can be accomplished by assessing the proportion of the P/L values of assets that exceed the VaR values of the proposed method. This section presents an analysis of the performance of the CredDGTN VaR approach utilizing Kupiec backtesting methodology (Kupiec 1995). The process of backtesting entails analyzing the frequency with which a risk metric is surpassed within a specified time interval. The backtesting outcomes of CredDGTN VaR for Portfolio I, Portfolio II, and Portfolio III, at the specified confidence levels, are displayed in Table 5 where NOL is the Number of Outliers and POL is the Percentage of Loss. As indicated in Table 5, the P-values of the CredDGTN VaR method were greater than (1-cl) at the specified confidence levels. Therefore, CredDGTN VaR can be considered a well-specified risk measure.

APPLICATION OF CREDIBLE DELTA-GAMMA (THETA)-NORMAL VALUE-AT-RISK USING REAL FINANCIAL DATA

The application of CredDGTN VaR in this section is employed to analyze the risk of Portfolio I-IV. The analyzed portfolio comprised stock options traded in the capital market. The stock options are similar to the options utilized in Sulistianingsih, Rosadi and

Abdurakhman (2023, 2021) to ease in comparing the performance of CredDGTN VaR to CredDGN VaR and CredDN VaR. The analyzed stock options are Advanced Micro Devices Inc (AMD), Bank of America Corp (BAC), Ford Motor Company (F), General Electric Company (GE), The Coca-Cola Company (KO), and Walmart Inc. (WMT). The six stock options were employed in four portfolios. Portfolio I was developed by KO and WMT; Portfolio II was constructed by GE, AMD, and F; Portfolio III consisted of AMD, BAC, KO, and WMT, while Portfolio IV consisted of AMD, BAC, GE, and F. The analyzed data ranged from July 23rd 2010 to July 23rd 2020. Firstly, the close stock prices underlying the analyzed option prices were divided into ten periods of risk analysis. The first period is from 1 July 2010 to 30 June 2011. The second period is from 1 July 2011 to 30 June 2012. Then, the third period until the tenth observed period also started on 1 July and ended on 30 June, where each observed period occupies one year. DGTNA VaR employed the second order of Taylor Polynomial to approximate the profit/loss of the option price so that the ten-period close price analyzed in this research was transformed into profit/loss data. Next, the quantification of DGTN VaR utilizing Equation (8) as well as its mean with confidence levels 0.80, 0.90, 0.95, and 0.99 for each analyzed asset and specified period was conducted. The process was assisted by constructing an R program to ease the complex calculation of the similar long recursive process in Sulistianingsih, Rosadi and Abdurakhman (2021). We just need to substitute DGN VaR with DGTN VaR in the process.

TABLE 1. $\sigma\ \Delta\ S_{t+\Delta t}$, Δt, and S_t of each asset

Option	σ	S_t	K
A	0.5	10	8
B	1	20	10
C	1.5	25	7
D	2	12	4

TABLE 2. Estimated Delta-Gamma (Theta)-Normal VaR of each asset

cl=80%				
Period (i)	$\eta^*_{i,A}$	$\eta^*_{i,B}$	$\eta^*_{i,C}$	$\eta^*_{i,D}$
1	0.434	0.883	1.333	1.667
2	0.374	0.861	1.305	1.708
3	0.424	0.792	1.274	1.722
4	0.458	0.831	1.262	1.673
5	0.437	0.845	1.255	1.634
6	0.425	0.890	1.197	1.810
7	0.415	0.840	1.277	1.620
8	0.410	0.875	1.201	1.760
9	0.434	0.817	1.315	1.683
10	0.427	0.886	1.257	1.630
$\widehat{E[\eta^*_j]}$	0.424	0.852	1.267	1.690

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cl=90%				
Period (*i*)	$\eta^*_{i,A}$	$\eta^*_{i,B}$	$\eta^*_{i,C}$	$\eta^*_{i,D}$
1	0.660	1.345	2.030	2.539
2	0.569	1.311	1.987	2.601
3	0.645	1.207	1.939	2.622
4	0.697	1.265	1.921	2.547
5	0.665	1.287	1.912	2.488
6	0.647	1.356	1.823	2.755
7	0.632	1.278	1.944	2.466
8	0.623	1.332	1.828	2.680
9	0.661	1.244	2.002	2.562
10	0.650	1.350	1.914	2.481
$\widehat{E[\eta^*_j]}$	0.646	1.298	1.930	2.574
cl=95%				
Period (*i*)	$\eta^*_{i,A}$	$\eta^*_{i,B}$	$\eta^*_{i,C}$	$\eta^*_{i,D}$
1	0.847	1.726	2.605	3.259
2	0.730	1.683	2.551	3.338
3	0.828	1.549	2.489	3.366
4	0.895	1.624	2.465	3.269
5	0.854	1.652	2.454	3.193
6	0.830	1.740	2.340	3.537
7	0.812	1.641	2.495	3.166
8	0.800	1.710	2.346	3.440
9	0.848	1.597	2.570	3.289
10	0.834	1.732	2.457	3.185
$\widehat{E[\eta^*_j]}$	0.829	1.666	2.477	3.303
cl=99%				
Period (*i*)	$\eta^*_{i,A}$	$\eta^*_{i,B}$	$\eta^*_{i,C}$	$\eta^*_{i,D}$
1	1.198	2.441	3.684	4.609
2	1.032	2.380	3.607	4.721
3	1.171	2.190	3.521	4.760
4	1.266	2.297	3.487	4.623
5	1.207	2.337	3.470	4.516
6	1.174	2.461	3.310	5.002
7	1.148	2.321	3.529	4.477
8	1.131	2.418	3.319	4.865
9	1.199	2.258	3.635	4.651
10	1.179	2.450	3.475	4.505
$\widehat{E[\eta^*_j]}$	1.172	2.356	3.503	4.672

TABLE 3. Estimators of $\widehat{\mu^*(v^*)}$, $E[\widehat{\tau^*(v^*)}]$, $Var[\widehat{\mu^*(v^*)}]$, and Z_{DGTN} at the specied confidence levels

	cl	$\widehat{\mu^*(v^*)}$	$E[\widehat{\tau^*(v^*)}]$	$Var[\widehat{\mu^*(v^*)}]$	$\widehat{Z_{DGTN}}$
Portfolio I	99%	1.763	0.006	0.701	0.999
	95%	1.247	0.003	0.351	0.999
	90%	0.971	0.002	0.213	0.999
	80%	0.638	0.001	0.092	0.999
Portfolio II	99%	2.343	0.009	1.360	0.999
	95%	1.657	0.004	0.680	0.999
	90%	1.291	0.003	0.413	0.999
	80%	0.848	0.001	0.178	0.999
Portfolio III	99%	2.926	0.014	2.263	0.999
	95%	2.069	0.007	1.131	0.999
	90%	1.612	0.004	0.687	0.999
	80%	1.058	0.002	0.296	0.999

TABLE 4. CredDGTN of each asset

Portfolio I				
	A	B		
$\Psi_{DGTN,99\%}$	1.171	2.355		
$\Psi_{DGTN,95\%}$	0.828	1.665		
$\Psi_{DGTN,90\%}$	0.645	1.297		
$\Psi_{DGTN,80\%}$	0.424	0.852		
Portfolio II				
	A	B	C	
$\Psi_{DGTN,99\%}$	1.171	2.355	3.503	
$\Psi_{DGTN,95\%}$	0.828	1.665	2.477	
$\Psi_{DGTN,90\%}$	0.645	1.297	1.930	
$\Psi_{DGTN,80\%}$	0.424	0.852	1.267	
Portfolio III				
	A	B	C	D
$\Psi_{DGTN,99\%}$	1.172	2.356	3.503	4.672
$\Psi_{DGTN,95\%}$	0.829	1.666	2.477	3.303
$\Psi_{DGTN,90\%}$	0.646	1.298	1.930	2.574
$\Psi_{DGTN,80\%}$	0.424	0.852	1.267	1.690

TABLE 5. Results of kupiec backtesting for CredDGTN

	Asset	cl(%)	NOL	POL	P-Value
Portfolio I	A	80	500	19.841	0.567
	B	80	4723	18.77	0.936
	A	90	250	9.921	0.536
	B	90	230	9.127	0.925
	A	95	123	4.881	0.585
	B	95	111	4.405	0.909
	A	99	29	1.151	0.192
	B	99	25	0.992	0.463
Portfolio II	A	80	500	19.842	0.567
	B	80	473	18.77	0.937
	C	80	511	20.278	0.353
	A	90	250	9.921	0.536
	B	90	230	9.127	0.925
	C	90	252	10	0.483
	A	95	122	4.841	0.621
	B	95	111	4.405	0.909
	C	95	132	5.238	0.273
	A	99	29	1.151	0.192
	B	99	25	0.992	0.463
	C	99	22	0.873	0.697
Portfolio III	A	80	500	19.841	0.567
	B	80	473	17.77	0.936
	C	80	511	20.278	0.353
	D	80	526	20.873	0.132
	A	90	250	9.921	0.536
	B	90	230	9.127	0.925
	C	90	252	10	0.483
	D	90	251	9.96	0.510
	A	95	122	4.841	0.621
	B	95	111	4.405	0.909
	C	95	132	5.238	0.273
	D	95	139	5.516	0.110
	A	99	29	1.151	0.192
	B	99	25	0.992	0.463
	C	99	22	0.873	0.697
	D	99	23	0.913	0.622

The estimated DGTN VaR, quantified by the information in Table 6, for GE, AMD, BAC, KO, and WMT over the ten periods at 80% confidence level are listed in Table 7. Meanwhile, the DGTN VaR of the five assets for 90%, 95%, and 99% confidence levels are also provided in Table 7 and can be calculated using the same procedures. The next step was to estimate the three parameters needed for the CredDGTN VaR computation for each asset in Portfolios I-IV using the information provided in Table 7. The estimated $\widehat{\mu^*(v^*)}$, $E\left[\widehat{\tau^*(v^*)}\right]$, and $Var\widehat{[\mu^*(v^*)]}$ of Portfolios I-IV are listed in Table 8. The findings presented in Table 8 indicate that, for Portfolios I, II, III, and IV, the mean maximum potential losses at an 80% confidence level for each asset were 0.622 dollars, 1.002 dollars, 0.637 dollars, and 0.852 dollars, respectively, relative to the asset price on a preceding day. Furthermore, the DGTN VaR variance means at an 80% confidence level were determined for each asset in Portfolio I, Portfolio II, Portfolio III, and Portfolio IV, yielding values of 0.099 dollars, 0.064 dollars, 0.078 dollars, and 0.052 dollars, respectively. Additionally, the variances of DGTN VaR mean at an 80% confidence level were calculated for Portfolio I, Portfolio II, Portfolio III, and Portfolio IV, resulting in values of 0.046, 0.729, 0.053, and 0.575, respectively.

The calculation of an estimated risk factor for CredDGTN VaR, denoted as $\widehat{Z_{DGTN}}$, using Equation (13), relies on the estimators of $\mu(v)$, $E[\tau^*(v^*)]$, and $Var[\mu^*(v^*)]$. Table 8 presents the estimators of $\mu^*(v^*)$, $E[\tau^*(v^*)]$, $Var[\mu^*(v^*)]$, and $\widehat{Z_{DGTN}}$ for confidence levels of 80%, 90%, 95%, and 99%. Furthermore, upon comparing the loss estimation for each asset using DGTN VaR as presented in Table 7 and $\widehat{\mu^*(v^*)}$ in Table 8, it is observed that the maximum potential loss estimation using CredDGTN VaR for the two assets in Portfolio I at designated confidence levels exhibited a tendency to converge towards the estimated mean of delta-gamma (theta)-normal VaR for each asset. The observed phenomenon can be attributed to the estimated risk factor of CredDGTN VaR, denoted as $\widehat{Z_{DGTN}}$, which has a relatively high value of approximately 0.850. According to the mathematical expression presented in Equation (12) of Theorem 1, the weight assigned to the estimated value of $\widehat{\mu^*(v^*)}$ is comparatively lower than the weight assigned to the average of DGTN VaR for each asset during the ten periods. The phenomenon of interest is also evident in Portfolio III, where the risk factor of CredDGTN VaR exhibits a value of approximately 0.850. The high credibility factor (0.850) observed in both Portfolio I and III at a 90% confidence level can be attributed to the estimated variance of DGTN VaR expectations. In other words, the portfolio exhibits a significant variance among its constituent assets, resulting in varying levels of risk associated with each asset in the portfolio.

CredDGTN VaR and its risk factor of the j^{th} asset for the four portfolios at various confidence levels can be calculated directly based on the preceding information using Equations (12) and (13), and the results are shown in Table 9. According to the results summarized in Table 9, the highest potential losses for the holder of Portfolio I in the following period of investment, when Portfolio I was held for one day with a confidence level of 80% for asset KO and WMT sequentially, were 0.483 dollars and 0.760 dollars. According to Table 9, the bigger the confidence level value, the greater the CredDGTN VaR value. As a result, the higher the stipulated level of confidence (cl), the larger the portfolio risk that investors must manage. Additionally, the amount of capital needed to cover investment losses increases with the level of confidence (cl) that is provided.

Similar to Sulistianingsih, Rosadi and Abdurakhman (2023, 2021), Kupiec Backtesting constructed by Kupiec (1995) is also employed to analyze the performance of CredDGTN VaR. The results of Kupiec Backtesting for CredDGTN VaR in Portfolio I-IV are listed in Table 10. NOL and POL in Table 10 successively are abbreviated as the Number of Outliers and Percentage of Loss. Table 10 presents the results of Kupiec backtesting of CredDGTN VaR at 80%, 90%, and 95% confidence levels. The results indicate that CredDGTN VaR was effective in assessing the risk. Meanwhile, the performance of CredDGTN VaR at a 99% confidence level was not effective in assessing the risk for this case. The results shown in Table 10 are in accordance with the research of Date and Bustreo (2016), who claimed that confidence levels higher than 95% are improper for risk measures based on Gaussian distribution. The performance is also coincident with CredDGN VaR and CredDN VaR, which are improper in assessing the asset risks at a 99% confidence level. Moreover, this research also indicates that the performance of CredDGTN VaR also outperforms the CredDN VaR and CredDGN VaR. This phenomenon can be suggested by quantifying the number of losses that are greater than each risk measure for a similar case.

TABLE 6. Mean, $\sigma_{\triangle S_{t+\triangle t}}$, S_t and K of each real asset's profit/loss

Asset	Mean	$\sigma_{\triangle S_{t+\triangle t}}$	S_t	K
GE	-0.003	0.289	7.060	1
AMD	0.017	0.624	61.790	30
BAC	0.004	0.388	24.310	15
F	-0.002	0.215	6.840	1
KO	0.007	0.478	48.160	39.5
WMT	0.028	1.041	130.70	65

TABLE 7. Estimated Delta-gamma (theta)-normal VaR of each asset

cl=80%					
Period (i)	$\eta^*_{i,GE}$	$\eta^*_{i,AMD}$	$\eta^*_{i,KO}$	$\eta^*_{i,BAC}$	$\eta^*_{i,WMT}$
1	1.655	0.983	0.673	0.348	0.384
2	2.041	0.784	0.355	0.348	0.561
3	1.602	1.145	0.361	0.328	0.561
4	1.614	1.103	0.393	0.454	0.463
5	1.946	0.983	0.357	0.439	0.695
6	2.453	1.132	0.363	0.418	0.813
7	1.641	0.375	0.403	0.320	0.571
8	2.167	0.355	0.357	0.346	1.137
9	1.745	0.849	0.425	0.368	0.949
10	2.188	1.314	0.850	0.699	1.758
$\widehat{E[\eta^*_j]}$	1.905	0.902	0.454	0.404	0.789
cl=90%					
Period (i)	$\eta^*_{i,GE}$	$\eta^*_{i,AMD}$	$\eta^*_{i,KO}$	$\eta^*_{i,BAC}$	$\eta^*_{i,WMT}$
1	2.520	1.224	0.852	0.472	0.570
2	3.107	0.984	0.522	0.459	0.854
3	2.440	1.421	0.540	0.580	0.855
4	2.458	1.368	0.542	0.563	0.704
5	2.964	1.219	0.531	0.541	1.058
6	3.735	1.406	0.544	0.458	1.238
7	2.498	0.566	0.550	0.459	0.869
8	3.299	0.518	0.518	0.521	1.731
9	2.660	1.292	0.646	0.559	1.444
10	3.311	2.001	0.654	0.568	1.200
$\widehat{E[\eta^*_j]}$	2.901	1.999	0.654	0.444	1.189

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cl=95%					
Period (i)	$\eta^*_{i,GE}$	$\eta^*_{i,AMD}$	$\eta^*_{i,KO}$	$\eta^*_{i,BAC}$	$\eta^*_{i,WMT}$
1	3.234	1.424	1.000	0.575	0.724
2	3.132	1.149	0.659	0.567	1.096
3	3.988	1.648	0.687	0.684	1.097
4	3.155	1.587	0.664	0.684	0.903
5	3.804	1.414	0.674	0.666	1.358
6	4.794	1.632	0.693	0.643	1.589
7	3.207	0.724	0.672	0.573	1.116
8	4.234	0.652	0.650	0.666	2.2221
9	3.410	1.659	0.829	0.716	1.854
10	4.275	2.568	1.660	1.366	3.436
$\widehat{E[\eta^*_j]}$	3.707	1.447	0.819	0.709	1.540
cl=99%					
Period (i)	$\eta^*_{i,GE}$	$\eta^*_{i,AMD}$	$\eta^*_{i,KO}$	$\eta^*_{i,BAC}$	$\eta^*_{i,WMT}$
1	4.574	1.799	1.277	0.768	1.012
2	4.403	1.459	0.916	0.770	1.550
3	4.429	2.076	0.964	0.879	1.552
4	4.462	1.998	0.893	0.859	1.277
5	5.380	1.779	0.943	0.833	1.921
6	6.780	2.057	0.973	0.787	2.248
7	4.535	1.020	0.900	0.789	1.578
8	5.989	0.903	0.899	0.938	3.142
9	4.823	2.346	1.172	1.010	2.622
10	6.047	3.632	2.348	1.932	4.890
$\widehat{E[\eta^*_j]}$	5.142	1.907	1.129	0.956	2.176

TABLE 8. Estimators of $\widehat{\mu(v)}$, $E[\widehat{\tau(v)}]$, $Var[\widehat{\mu^*(v^*)}]$, and Z_{DGTN} at the specified confidence levels

cl	$\widehat{\mu^*(v^*)}$	$E[\widehat{\tau^*(v^*)}]$	$Var[\widehat{\mu^*(v^*)}]$	$\widehat{Z_{DGTN}}$
Portfolio I				
99%	1.652	0.751	0.473	0.863
95%	1.179	0.374	0.222	0.856
90%	0.927	0.227	0.127	0.848
80%	0.622	0.099	0.046	0.825
Portfolio II				
99%	2.520	0.442	5.602	0.992
95%	1.845	0.213	2.915	0.993
90%	1.464	0.132	1.742	0.992
80%	1.002	0.064	0.729	0.991

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Portfolio III				
99%	1.542	0.552	0.295	0.842
95%	1.127	0.275	0.155	0.849
90%	0.905	0.168	0.100	0.856
80%	0.637	0.078	0.053	0.870
Portfolio IV				
99%	2.129	0.362	4.339	0.992
95%	1.560	0.174	2.266	0.992
90%	1.240	0.107	1.360	0.992
80%	0.852	0.052	0.575	0.991

TABLE 9. CredDGTN of each asset

Portfolio I	CredDGTN	KO	WMT		
	$\Psi_{DGTN,80\%}$	0.483	0.760		
	$\Psi_{DGTN,90\%}$	0.695	1.159		
	$\Psi_{DGTN,95\%}$	0.871	1.488		
	$\Psi_{DGTN,99\%}$	1.200	2.105		
Portfolio II	Asset	GE	AMD	F	
	$\Psi_{DGTN,80\%}$	1.897	0.903	0.206	
	$\Psi_{DGTN,90\%}$	2.890	1.202	0.300	
	$\Psi_{DGTN,95\%}$	3.710	1.449	0.378	
	$\Psi_{DGTN,99\%}$	5.122	1.912	0.526	
Portfolio III	Asset	BAC	AMD	KO	WMT
	$\Psi_{DGTN,80\%}$	0.434	0.868	0.478	0.769
	$\Psi_{DGTN,90\%}$	0.616	0.157	0.690	1.158
	$\Psi_{DGTN,95\%}$	0.767	1.398	0.865	1.477
	$\Psi_{DGTN,99\%}$	1.049	1.849	1.194	2.076
Portfolio IV	Asset	BAC	AMD	F	GE
	$\Psi_{DGTN,80\%}$	0.408	0.902	0.204	1.896
	$\Psi_{DGTN,90\%}$	0.573	1.200	0.298	2.888
	$\Psi_{DGTN,95\%}$	0.709	1.447	0.376	3.707
	$\Psi_{DGTN,99\%}$	0.966	1.909	0.524	5.117

TABLE 10. Results of kupiec backtesting for CredDGTN

	Asset	cl(%)	NOL	POL	P-Value
Portfolio I	KO	80	219	8.708	1
	WMT	80	297	11.809	1
	KO	90	113	4.493	1
	WMT	90	152	6.044	1
	KO	95	63	2.505	1
	WMT	95	103	4.095	0.981
	KO	99	35	1.392	0.024
	WMT	99	46	1.829	0.000
Portfolio II	GE	80	399	15.865	1
	AMD	80	75	2.982	1
	F	80	307	12.207	1
	GE	90	211	8.390	0.997
	AMD	90	56	2.227	1
	F	90	167	6.640	1
	GE	99	50	1.988	0.000
	AMD	99	26	1.034	0.382
	F	99	48	1.909	0.000
Portfolio III	BAC	80	175	6.958	1
	AMD	80	81	3.221	1
	KO	80	223	8.867	1
	WMT	80	295	11.730	1
	BAC	90	101	4.016	1
	AMD	90	57	2.266	1
	KO	90	113	4.493	1
	WMT	90	152	6.044	1
	BAC	95	62	2.465	1
	AMD	95	52	2.068	1
	KO	95	65	2.584	1
	WMT	95	104	4.135	0.976
	BAC	99	34	1.352	0.036
	AMD	99	29	1.153	0.189
	KO	99	35	1.392	0.024
	WMT	99	48	1.909	0.000
Portfolio IV	BAC	80	199	7.913	1
	AMD	80	75	2.982	1
	F	80	307	12.207	1
	GE	80	399	15.865	1
	BAC	90	113	4.493	1
	AMD	90	56	2.227	1
	F	90	167	6.640	1
	GE	90	211	8.390	0.997
	BAC	95	80	3.181	1
	AMD	95	51	2.028	1
	F	95	105	4.175	0.971
	GE	95	122	4.851	0.612
	BAC	99	41	1.630	0.001
	AMD	99	26	1.034	0.382
	F	99	48	1.909	0.000
	GE	99	51	2.028	0.000

CONCLUSION

Credible Delta Gamma (Theta) Normal VaR was constructed to complete the previous risk measures, namely credible VaR, credible Delta Normal VaR, and Credible Delta Gamma Normal VaR. CredDGTN provides more Greek rather than CredDN and CredDGN VaR, namely Theta, to estimate the risk of a portfolio comprised of European call options. The performance of CredDGTN occupied by Kupiec Backtesting suggested that the risk measure is effective in measuring the risk of option portfolios even when the option returns are non-normally distributed. Furthermore, among CredDN, CredDGN, and CredDGTN, the performance of CredDGTN is considered the best one because the number of losses that are greater than CredDGTN is the smallest compared to the greater number of losses from CredDN and CredDGN in the same case. Therefore, CredDGTN can be considered as a proper alternative risk measure to evaluate the option risk. Then, for future research, it can be considered to add other option Greeks, namely vega and rho to complete and strengthen the existing methods.

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