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Modeling Population Harvesting of Rodents for the Control of Hantavirus Infection

(Pemodelan Proses Populasi Penuaian Tikus bagi Mengawal Jangkitan Hantavirus)

F.M. YUSOF*, A.I.B. MD. ISMAIL & N.M. ALI

ABSTRACT

Hantaviruses are infectious agents that can cause diseases resulting in deaths in humans and are hosted by rodents without affecting the hosts themselves. A simple mathematical model describing the spread of the Hantavirus infection in rodents has been proposed and developed by Abramson and Kenkre where the model takes into account the temporal and spatial characteristics of this infection. In this paper, we extended this model to include the process of harvesting and study the impact of different harvesting strategies in the spread of the Hantavirus infection in rodents. Several numerical simulations were carried out and the results are discussed.

Keywords: Hantavirus; harvesting; mathematical model; numerical simulations

ABSTRAK

Hantavirus adalah ejen jangkitan penyakit yang boleh menyebabkan kematian di kalangan manusia dan berperumahkan tikus tanpa memberi kesan kepada perumah itu sendiri. Model matematik mudah yang menjelaskan pembiakan jangkitan hantavirus ke atas tikus telah dicadang dan dibangunkan oleh Abramson dan Kenkre dengan model tersebut mengambilkira ciri ruang dan masa jangkitan ini. Dalam makalah ini, kami meluaskan model ini dengan memasukkan proses penuaian dan mengkaji kesan strategi penuaian yang berbeza ke atas pembiakan jangkitan hantavirus ke atas tikus. Beberapa simulasi berangka telah dijalankan dan keputusan dibincangkan.

Kata kunci: Hantavirus; penuaian; model matematik; simulasi berangka

INTRODUCTION

Hantaviruses are a group of viruses that are carried by certain kinds of rodents. Two major diseases caused by hantaviruses are hemorrhagic fever with renal syndrome (HFRS) and hantavirus cardiopulmonary syndrome (HPS) (Hjelle 2007). Hantaviruses are hosted by rodents, such as rats and mice, without causing any adverse effects to the hosts themselves. The viruses can be transmitted to humans by way of human contact with the urine, feces or saliva from the infected rodents.

In 1993, an outbreak of HPS occurred in the South West USA resulting in a high mortality rate. A basic mathematical model was developed by Abramson and Kenkre (2002) to simulate the spread of the virus and was found to be able to replicate some features of the infection such as the sporadic disappearance of the infection and the existence of refugias for the rodents when environmental conditions are not favourable for the rodents (lack of water, food, and shelter).

Since the pioneering work of Abramson and Kenkre (2002), there have been several other studies of the spread of Hantavirus infection using mathematical models. The dynamics of simple traveling wave as a mechanism for propagating Hantavirus infection has been studied by Abramson et al. (2003). Piexoto and Abramson (2006) studied the effect of biodiversity on the spread of Hantavirus infection. In particular they studied the effect of the existence of other species competing for resources with rodents. Giuggioli et al. (2006) studied the spread of Hantavirus infection by incorporating into the basic model data regarding the movement of localised adult and itinerant juvenile rodents. There have also been some studies on the use of the Variational Iteration Method to solve the basic governing equations of Abramson and Kenkre (Goh et al. 2009) and the use of cellular automata to simulate the spread of Hantavirus infection (Abdul Karim et al. 2009).

Population harvesting is defined as the removal of a constant number of individual from a population during each time period (Miner & Wicklin, 1996). Such a policy has been used to stabilise population in an environment with limited resources or carrying capacity. There have been recent work on population harvesting by Bairagi et al. (2009) and Matsuoka and Seno (2008). According to Bairagi et al. (2009), epidemiology can encroach into ecology and change the system dynamics significantly. In population ecology, predator-prey interaction in presence of parasites can produce more complex dynamics including switching of stability, extinction and oscillations. Bairagi et al. (2009) states that harvesting can play a crucial role in a host-parasite system and reasonable harvesting can remove a parasite from their host. In their paper, the role of harvesting in a predator-prey-parasite system has been studied. Their study shows that impulsive harvesting can control the cyclic behaviour of the system populations leading to the persistence of all species and obtain disease-free stable equilibrium.

Matsuoka and Seno (2008) analysed a time-discrete mathematical model of host-parasite population dynamics with harvesting, in which the host can be regarded as a pest. A portion of the host population is harvested at a moment in each parasitism season with the principal target being the host. However, the parasite population may also be affected and reduced by a portion. They investigate the condition under which the harvesting of the host results in an eventual increase of its equilibrium population size.

In the light of these studies, there seems to be a basis to study harvesting of rodents as a strategy to control Hantavirus infection. In this paper we develop suitable mathematical models and conduct numerical experiments to study the effects of population harvesting of rodents on the spread of Hantavirus infection.

Mathematical Modeling of the Hantavirus Infection

Consider the basic model of Abramson and Kenkre (2002) which is of the form

$$\frac{dr_s}{dt} = br - cr_s - \frac{r_s r}{k} - ar_s r_i \quad \text{and}$$
$$\frac{dr_i}{dt} = -cr_i - \frac{r_i r}{k} - ar_s r_i \tag{1}$$

where r_s and r_i are the populations of susceptible and infected rodents, respectively, where $r(t) = r_s(t) + r_i(t)$ is the total population of rodents. For abbreviation, we shall refer to this model as the basic *AK* model.

The value br represents the births of rodents, all of them born vulnerable to the infection at a rate proportional to the total population assuming that all rodents contribute equally to the reproduction process. The value c represents the natural death rate. The infection does not cause deaths among rodents. The value $-\frac{r_{s,i}r}{k}$ represents a limitation process in the rodent population growth due to competition for resources shared between r_s and r_i. In the basic model, parameter k depends on time and is a "environmental parameter". Higher values of k represent higher availability of water, food, shelter and other resources for the rodents. $ar_s r_i$ represents the number of susceptible rodents that get infected due to an encounter with an infected rodent (e.g. bites from fights) at a rate a (assumed constant). The value a is known as the "aggression parameter".

According to Abramson and Kenkre (2002), there is a critical value of the environmental parameter $\left(k_c = \frac{a}{a(b-c)}\right)$ that separates two distinctive regimes. If the environmental parameter k is smaller than k_c, r_i tends to zero and the infection dies away. If $k > k_c$, the infection thrives since there is an increase in resources. As the environmental parameter will vary with time, the system will undergo transitions from one state to another.

The basic model can be spatially extended to take into account the movement of rodents by including a diffusive term as follow:

$$\frac{\partial r_s}{\partial t} = br - cr_s - \frac{r_s r}{k(x,y,t)} - ar_s r_i + D\nabla^2 r_s$$
$$\frac{\partial r_i}{\partial t} = -cr_i - \frac{r_i r}{k(x,y,t)} - ar_s r_i + D\nabla^2 r_i \tag{2}$$

where r_s and r_i are the populations of susceptible and infected rodents, respectively, and $r(t) = r_s(t) + r_i(t)$ is the total population of rodents.

The value D is the diffusion coefficient. The analysis by Abramson and Kenkre (2002) for small and moderate values of the diffusion coefficient shows that the infected population survives in the regions of high environmental parameter and becoming extinct in the rest. These "islands" of infection become reservoirs or refugia of the virus and it is from these locations that the disease will spread when environmental conditions become favourable.

Harvesting Applied to Rodents

In this paper, we consider three types of harvesting models to be incorporated into the basic *AK* model. These are the constant harvesting model, seasonal (periodic) harvesting model and proportional harvesting model (Idels & Wang 2008). The details of each harvesting model implemented on the rodents population are given as follows:

Constant Harvesting

Constant harvesting involves the removal of a fixed number of rodents for each time period. We assume that a constant number, h, of rodents are removed from the population. Therefore, the model is given by

$$\frac{\partial r_s}{\partial t} = br - cr_s - \frac{r_s r}{k} - ar_s r_i - h$$
$$\frac{\partial r_i}{\partial t} = -cr_i - \frac{r_i r}{k} + ar_s r_i - h \tag{3}$$

Seasonal (Periodic) Harvesting

The equations

$$\frac{\partial r_s}{\partial t} = br - cr_s - \frac{r_s r}{k} - ar_s r_i - h\left(1+\sin\left(wt\right)\right)$$
$$\frac{\partial r_i}{\partial t} = -cr_i - \frac{r_i r}{k} + ar_s r_i - h\left(1+\sin\left(wt\right)\right) \tag{4}$$

represent a seasonal or periodic harvesting model. The parameter h represents the coefficient that determines the total rate of periodic harvesting and w represents the wavelength of the sinusoidal function that dictates periodic harvesting (Cross et al. 1998).

PROPORTIONAL (VARIABLE) HARVESTING

The equations are:

$$\frac{\partial r_s}{\partial t} = br - cr_s - \frac{r_s r}{k} + ar_s r_i - Er_s \quad \text{and}$$
$$\frac{\partial r_i}{\partial t} = -cr_i - \frac{r_i r}{k} + ar_s r_i - Er_i \tag{5}$$

where E represents the fraction of the population removed for each time period. It can considered as the harvesting effort.

NUMERICAL EXPERIMENTS AND DISCUSSION OF RESULTS

We solve the systems (1), (3), (4), (5) numerically to simulate the effects of the different harvesting strategies mentioned in Section 3 on the rodents population. The Matlab software ODE45 was used in all of our numerical experiments. ODE45 is based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair and it is a one-step solver. Model parameters used by Abramson and Kenkre (2002) were used in the experiments, viz. a = 01, b = 1, c = 0.5. Hence, k_c = 20. The simulation results are shown over a period of 20 years with the environmental parameter k exceeding the critical value k_c. The value k = 80 and k = 5 are used which means the environmental conditions are favourable and thus the infection is thriving.

Figure 1 shows the rodent population for the case k(= 80)>k_c when the basic *AK* model is solved using the same initial values (= 50) for r_s and r_i.

The behavior of the system is quite independent of the choice of initial condition. We have tested this via numerical experiments. 50 was chosen because it was the value chosen by some previous students who studied the problem. For this model, the abundance of resources such as water and food at the initial stage will cause the infected population to increase sharply initially and reaches a certain maximum before plunging down and stabilise at a steady value of 45. Since the infection was thriving initially, this reduces the population of susceptible rodents drastically before rising slightly and approaching a stable value when the infected numbers starts to stabilise. After 4 years, the susceptible population r_s will eventually stabilise at a steady value of 12. Thus, both population groups will eventually reach steady-states with the values for infected population being higher compared to the susceptible case as predicted by theory. Figures 2, 3 and 4 display the population of rodents for constant with h = 10.0, seasonal $\left(\text{with } h = 10.0 \text{ and } w = \frac{\pi}{300}\right)$ and proportional with $E = 0.9$ harvesting models, respectively. In all of these cases, the same initial values for r_i, r_s and k are used as in the basic *AK* model with no harvesting.

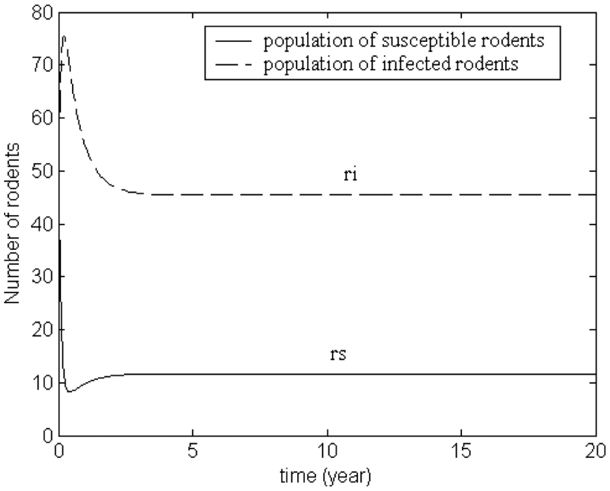

FIGURE 1. Values of r_s and r_i for basic *AK* model with initial values r_s = 50, r_i = 50 and k = 80

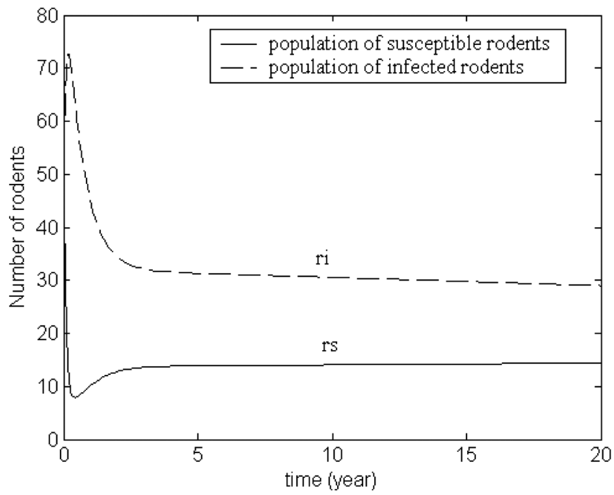

FIGURE 2. Values of r_s and r_i for constant harvesting with initial values r_s = 50, r_i = 50 and k = 80, (h = 10.0)

For the constant harvesting model, steady values of 32 and 14 were reached by r_i and r_s respectively while for the seasonal model, the infected and susceptible population will stabilize to steady values of 29 and 15 respectively. The proportional harvesting model has the smallest steady-state value for r_i and the largest susceptible steady value r_s, i.e. 20 and 18 respectively. In all of the cases above, the steady values for r_i are always higher than the steady values of r_s. What is important to note here is that the infection persists. This is due to the abundant resources available over the 20 years period (very high value of k).

The three harvesting models display similar graphical pattern as the basic *AK* model with no harvesting but with varying values of r_i and r_s. With $k > k_c$, the infected population increase initially due to the more abundant resources that the rodents can use to thrive. It then reaches

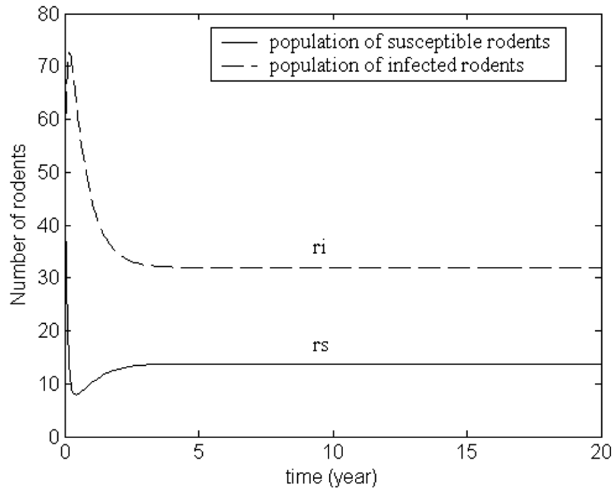

FIGURE 3. Values of r_s and r_i for seasonal harvesting with initial values $rs = 50$, $r_i = 50$ and $k = 80$, ($h = 10.0$ and $w = \frac{\pi}{300}$)

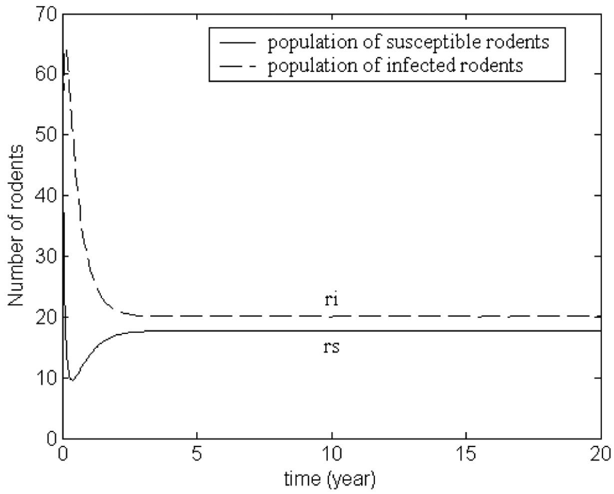

FIGURE 4. Values of r_s and r_i for proportional harvesting with initial values $r_s = 50$, $r_i = 50$ and $k = 80$, ($E = 0.9$)

a certain maximum after which it starts to decrease most probably due to the resources being almost used up during the peak population. Meanwhile the susceptible rodent population behave in the opposite way which is quite expected since more resources means more rodents are being infected and thus lessen the number of susceptible. Both types of population will then reach steady stable values over some period of time with the number of infected rodents always exceeding the number of susceptible rodents. The steady state values of infected rodents are always less than the corresponding values in *AK* model with no harvesting (as one would expect). The steady state of values of susceptible rodents is always slightly higher in the harvesting cases compared with the corresponding value in the basic *AK* model. Thus, the intensity if the infection has been reduced by harvesting with the reduction in intensity being most pronounced for proportional harvesting. Tests with other values of *h, w* and *E* indicate the continued persistence of the infection but with different performances for the harvesting strategy adapted.

Figure 5 shows the rodent population for the case k (= 55) > k_c when the basic *AK* model with no harvesting is solved using the same initial values (= 50) for r_s and r_i.

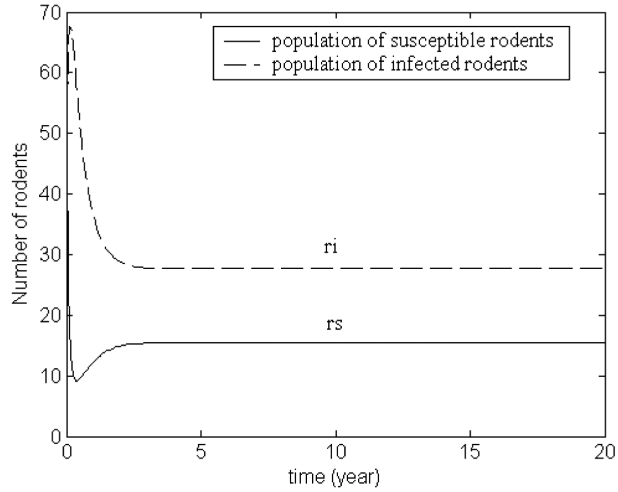

FIGURE 5. Values of r_s and r_i for basic *AK* model with initial values $r_s = 50$, $r_i = 50$ and $k = 55$

For this model, the abundance of resources (albeit less than in the previous use of $k = 55$) such as water and food at the initial stage will cause the infected population to increase sharply initially and reaches a certain maximum before plunging down and stabilize at a steady value of 28. Since the infection was thriving initially, this reduces the population of susceptible rodents drastically before rising slightly and approaching a stable value when the infected numbers starts to stabilize. After 4 years, the susceptible population r_s will eventually stabilize at a steady value of 15. In any case, both population groups will eventually reach steady-states with the values for infected population being higher compared to the susceptible case. Figures 6, 7 and 8 display the population of rodents for constant with $h = 10.0$, seasonal and proportional with $E = 0.9$ harvesting models respectively. In all of these cases, the same initial values for r_i, r_s and k (= 55) are used as in the basic *AK* model with no harvesting.

For the constant harvesting model, steady values of 15 and 20 were reached by r_i and r_s respectively while for the proportional model, the infected and susceptible population will stabilize to steady values of 10 and 22 respectively. The seasonal harvesting model does not attain a steady-state for r_i and r_s. In all of the cases above, the values for r_s are always higher than the values of r_i. What is important to note here is that the infection has subsided due to harvesting.

Tests with other values of *h, w* and *E* indicate the subsidence of the infection but with different performances

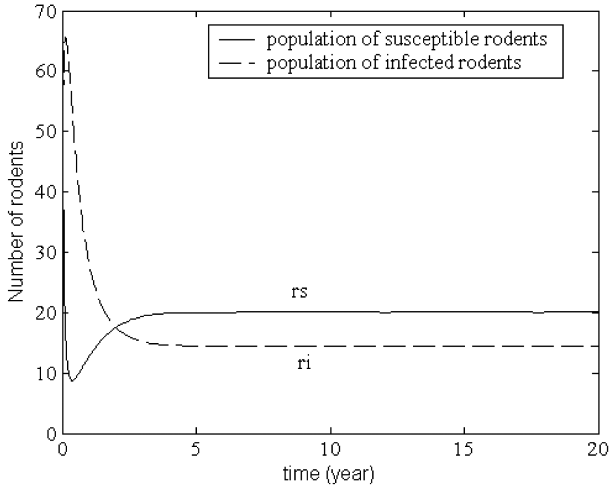

FIGURE 6. Values of r_s and r_i for constant harvesting with initial values $r_s = 50$, $r_i = 50$ and $k = 55$, ($h = 10.0$)

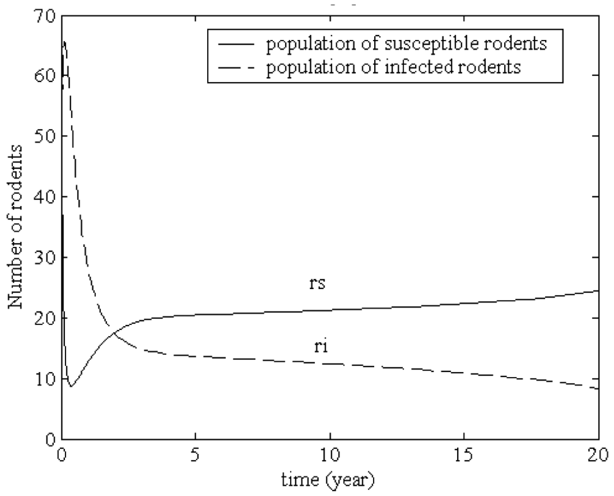

FIGURE 7. Values of r_s and r_i for seasonal harvesting with initial values $r_s = 50$, $r_i = 50$ and $k = 55$, ($h = 10.0$ and $w = \frac{\pi}{300}$)

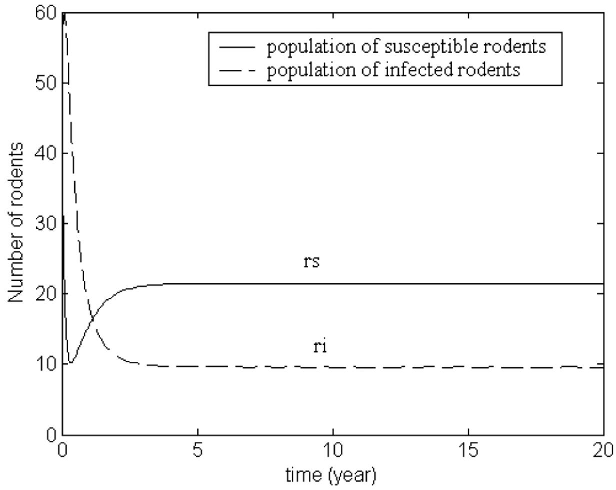

FIGURE 8. Values of r_s and r_i for proportional harvesting with initial values $r_s = 50$, $r_i = 50$ $k = 80$, ($E = 0.9$)

for the harvesting strategy adopted. However it can be expected that certain choices will result in the persistence of the infection.

CONCLUSION

In this work, we study the effects of population harvesting on the spread of Hantavirus infection. We extend the model developed by Abramson and Kenkre (2002) by incorporating several types of population harvesting processes into the model and we study how the demise of certain number of rodents will affect the spread of hantavirus infection. In particular we study the evolution of the infected population of rodents, r_i, together with the susceptible population, r_s, over a period of time when higher resources are available. For situations where abundant resources are available population harvesting may reduce the intensity of the infection but is not able to control it. However when resources are only moderately abundant, population harvesting can ensure that the infection subsides.

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School of Mathematical Sciences
Universiti Sains Malaysia
11800 USM, Penang
Malaysia

*Corresponding author; email: fauzikmk@hotmail.com

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